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THE PROBABILISTIC ARGUMENT FROM EVIL

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I. INTRODUCTION: WHAT, EXACTLY, IS THE PROBLEM?

Ever since the days of Epicurus there have been philosophers who believed that the existence of evil constitutes a formidable objection to theistic belief and a powerful argument for atheism. We might call those who urge this argument 'natural atheologians'; just as the natural theologian offers arguments for the existence of God, or for the rational propriety of theistic belief, so the natural atheologian offers arguments for the non-existence of God, or for the rational impropriety of theistic belief. The vast majority of those who offer an atheological argument from evil have held that the existence of evil (or of the amount and kind we find) is *inconsistent* with the existence of a wholly good, omniscient and omnipotent God.[1] So, for example, McCloskey:

> Evil is a problem, for the theist, in that a *contradiction* is involved in the fact of evil on the one hand and belief in the omnipotence and omniscience of God on the other.[2]

And Walter Kaufmann:

> Once these assumptions [that God is good, just, and omnipotent] are granted, the problem arises: why, then, is there all the suffering we know? And as long as these assumptions are granted, this question cannot be answered. For if these assumptions were true, it would follow that there could not be all of this suffering. Conversely: since it is a fact that there is all this suffering, it is plain that at least one of these assumptions must be false. Popular theism is refuted by the existence of so much suffering. The theism preached from thousands of pulpits and credited by millions of believers is disproved by Auschwitz and a billion lesser evils.[3]

More recently, however, those who make an atheological argument from evil have conceded that the existence of evil may be logically consistent with that of God; nevertheless, the former renders the latter unlikely, or improbable, or provides evidence against it, or gives a good reason for accepting its denial.[4] This is certainly a change for the better; it can be shown, I think, that the existence of evil is *not* logically inconsistent with God's existence. I have argued elsewhere[5] that the proposition

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G God exists and is omniscient, omnipotent and wholly good

is compatible with

E There are 10^{13} turps of evil

where the *turp* is the basic unit of evil, so that '10^{13} turps' is a name of the evil, past, present and future, the actual world (call it 'α') contains. G and E are consistent, I argued, in the sense that there are possible worlds in which both are true. Pared to bare essentials, the argument proceeded by pointing out that G seems to be consistent with

(1) God is the omnipotent, omniscient and wholly good creator of the world; and every world God could have actualized that contains less than 10^{13} turps of evil, contains less good and a less favorable overall balance of good and evil than the actual world contains.[6]

But the conjunction of G and (1) entials E; hence by a familiar principle of modal logic, G is consistent with E.

Let's agree then, at least for purposes of argument, that G is consistent with E. Now the probabilistic argument contends that E is evidence against G, or that given E, it is unlikely that G is true. And we shall have to ask why the atheologian thinks this is so; what is his reason for thinking G *is* improbable with respect to E? But before we ask that question, let's suppose, for the moment, that the probabilistic atheologian is right and that the fact is, G *is* improbable on E. What is supposed to follow from that? How is that to be construed as an objection to theistic belief? How does the atheologian's argument go from there? It doesn't follow, of course, that theism is false. Nor does it follow that one who accepts both G and E (and, let's add, recognizes that G is improbable with respect to E) has an irrational system of beliefs or is in any way guilty of noetic impropriety. For it could be, of course, that G is improbable with respect to E but probable with respect to something else we know. I might know, for example, both that

(2) Feike is a Frisian and 9 out of 10 Frisians can't swim,

and

(3) Feike is a Frisian lifeguard and 99 out of 100 Frisian lifeguards can swim;

it is plausible to hold that

(4) Feike can swim

is probable with respect to (3) but improbable with respect to (2). If, furthermore, (2) and (3) are all we know about Feike's swimming ability, the view that he can swim is more acceptable than the view that he can't, even though we know something with respect to which the former is improbable. Indeed, we might very well *know* both (2) and (4); we might very well know a pair of propositions *A* and *B* such that *A* is improbable on *B*. So even if it were a fact that E is evidence against G or that G is improbable on E, that fact, so far, isn't of much consequence.

What the atheologian must show, if he wants to show that there is a viable objection to theism here, is that on some relevant body of *total* evidence – his own, perhaps, or the theist's, or perhaps a body of total evidence shared by all those who are party to the dispute – G is improbable. If he could show that G is improbable on his *own* total evidence, the atheologian could show that *he* has a good reason for rejecting theism; if he could show that G is improbable on the *theist's* total evidence then he could show, perhaps, that the *theist* is irrational or guilty of noetic impropriety in accepting G. It won't be of any use, incidentally, for him to show that G is improbable on what the theist and atheologian *both* know – i.e., the set of propositions *S* such that *A* is a member of *S* if and only if both theist and atheologian know *A*. Quantum mechanics, no doubt, is quite improbable with respect to what my mother and Richard Feynman both know; but that's no objection to it.

So the atheologian must show that G is improbable with respect to the relevant body of total evidence, whatever exactly that is. To do this, he would be obliged to consider all the sorts of reasons natural theologians have invoked *in favor of* theistic belief – the traditional cosmological, teleological and ontological arguments, for example. He would also have to consider more recent versions of the moral argument as developed, for example, by A. E. Taylor, and still more recently by Robert Adams, along with the sorts of broadly inductive arguments developed by F. R. Tennant, C. S. Lewis, E. L. Mascall, Basil Mitchell, and others; and he'd have to show either that these arguments don't really produce any evidence for G at all, or that, if they do, that evidence is outweighed by the evidence *against* G furnished by E. This would be a substantial and difficult project – one no atheologian has undertaken so far. Herein lies the beauty of the deductive argument from evil

(from the atheological point of view); if that argument could only be made to work the putative favorable evidence could be gracefully ignored.

Perhaps we may best understand the atheologian as purveying a sort of challenge: G is improbable on E; so the theist must concede that G is improbable on the total evidence, unless he can show that G is probable on T – E, the relevent body of total evidence apart from E. So if the theist can't establish any of the arguments *for* G, he should concede that G is improbable on total evidence. (And, so far, the atheologian might add, none of the theistic arguments looks very compelling.) Here the alleged principle underlying the challenge seems to be

(5) If *A* is improbable with respect to *B* and also with respect to *C*, then *A* is improbable with respect to *B and C*.

Of course the theist may not be at all inclined to agree that G *is* improbable with respect to T – E, even if he can't show that it is probable with respect to that body of evidence; things of this sort are notoriously difficult to demonstrate. But in any event the 'principle' on which the challenge is based is false. Consider a modest lottery of 8 tickets. That either ticket 2 or ticket 3 will win is improbable on the information that one of the first five tickets will win, as it is on the information that one of tickets 2, 3, 4, 7 and 8 will win. On the conjunction of these bits of information, however, it *is* probable that either ticket 2 or ticket 3 will win.

A great deal more needs to be said, therefore, about what is supposed to follow from the alleged fact that G is improbable with respect to E. But suppose we concede, for purposes of argument, that if G is improbable on E, then it is improbable on the relevant body of total evidence, whatever exactly that may be. This is an enormous assumption; there seems little if any reason to think it true; but I shall not presently contest it.

Our central question, then: *is* G improbable with respect to E? It isn't self-evident or obvious that it is; so why should we think so? A theist, after all, might accept the ontological argument, or like Aquinas, reject the argument but accept its conclusion: that God exists necessarily, exists in every possible world and is essentially omniscient, omnipotent and wholly good. But if this is so, then G is a necessary truth, in which case E (and every other proposition) entails G. But if E entails G, then the denial of G is not probable with respect to E – not, at any rate, if the axioms of the probability calculus govern the relationship the atheologian claims G and E stand in, when he

claims that G is improbable with respect to E. In holding that G is improbable on E, therefore, the atheologian is committed to holding that no version of the ontological argument is sound and that its conclusion is false. He is not committed merely to holding that the Ontological Argument is not a *proof* of its conclusion, or that it is not a successful piece of natural theology; he must hold that the argument is unsound and its conclusion *false*. True enough, the atheologian may not *mind* being thus committed; but if he has no argument for the claim that G is improbable on E, it is hard to see how how he can sensibly accuse the theist of noetic impropriety in accepting both G and E. It would be as if the theist, in the absence of any argument whatever, were to claim that the atheologian is irrational in holding, as he does, that G is improbable on E, when the fact is G is a necessary truth and hence has a probability of 1 with respect to E. So the atheologian needs an argument for his claim that G is improbable on E.

And even if we hold in abeyance the issue of God's necessary existence, we must ask what reason there is for thinking G is improbable on E. We have agreed that G and E are consistent; it *could* be that God could not have actualized a world as good as this one with less evil. On what grounds, then, are we to hold that though G and E are consistent, G is improbable on E? Surely the atheologian needs an argument here. If he simply *asserts* this claim the theist can simply retort that he doesn't think so.

II. ARGUMENTS FOR *G IS IMPROBABLE ON E*

What kind of argument could he give? It isn't easy to find published argument for this conclusion; Cornman and Lehrer, however, produce a sort of argument:

> If you were all-good, all-knowing, and all-powerful, and you were going to create a universe in which there were sentient beings – beings that are happy and sad, enjoy pleasure, feel pain, express love, anger, pity, hatred – what kind of world would you create? Try to imagine what such a world would be like. Would it be like the one which actually does exist, this world we live in? Would you create a world such as this one if you had the power and know-how to create any logically possible world? If your answer is 'no', as it seems it must be, then you should begin to understand why the evil of suffering and pain in this world is such a problem for anyone who thinks God created this world; then, it seems, we should conclude that it is improbable that it was created or sustained by anything we would call God. Thus, given this particular world, it seems we should conclude that it is improbable that God – who if he exists, created the world – exists. Consequently, the belief that God does not exist, rather than the belief that he exists, would seem to be justified by the evidence we find in this world.[7]

This appears to be an argument for the dual conclusion that G is improbable on E and also improbable on the relevant body of total evidence. And while its exact structure may not be wholly clear, it is clear that the argument is vitiated by it's reliance on 'Leibniz' Lapse' – the idea that if God is omnipotent, then it follows that He could have actualized just any possible world.[8] And in any event is there really more, here, than the simple assertion that G is indeed improbable on E – and on the relevant total evidence?

In the course of developing what he calls a 'strong argument' for atheism, William Rowe gives an argument whose conclusion, I think, is that E is evidence for the denial of G:

> ...we must then ask whether it is reasonable to believe that all the instances of profound, seemingly pointless human and animal suffering lead to greater goods. And, if they should somehow all lead to greater goods, is it reasonable to believe that an omnipotent, omniscient being could not have brought about any of those goods without permitting the instances of suffering which supposedly lead to them? When we consider these more general questions in the light of our experience and knowledge of the variety and profusion of human and animal suffering occurring daily in our world, it seems that the answer must be no. It seems quite unlikely that all the instances of intense human and animal suffering occurring daily in our world lead to greater goods, and even more unlikely that if they all do, an omnipotent, omniscient being could not have achieved at least some of those goods without permitting the instances of suffering that lead to them. In the light of our experience and knowledge of the variety and scale of human and animal suffering in our world, the idea that none of those instances of suffering could have been prevented by an omnipotent being without the loss of a greater good seems an extraordinary, absurd idea, quite beyond our belief.[9]

Here Rowe suggests that the theist is committed to a double supposition:

(6) All instances of human and animal suffering 'lead to' greater goods,

and

(7) God, who is omniscient and omnipotent, could not have brought about *any* of these goods without permitting the evils that lead to them.

I'm not sure how to understand 'leads to' as Rowe uses it here; perhaps his idea is that (according to the theist) each instance of suffering is a *counterfactually sufficient condition* of a greater good. This is unimportant, however; what really counts is the second part of what Rowe attributes to the theist: that God could not have brought about certain good states of affairs that do in fact obtain (the greater goods to which the instances of suffering lead) without permitting those instances of suffering. Apparently Rowe believes

the theist is committed to holding that for each instance of suffering there is a greater good God could not have brought about without permitting that instance of suffering – a greater good being *greater* in that it outweighs the evil of the instance of suffering. And perhaps we can say that a good state of affairs G *outweighs* an evil state of affairs E if and only if the conjunctive state of affairs G *and* E is a good state of affairs. As Rowe sees it, then, the theist is committed to

(8) For each evil state of affairs E that obtains, there is a good state of affairs G such that (a) G outweighs E, and (b) God, though omnipotent and omniscient, could not have brought about G without permitting E.

This is the first step of Rowe's argument. The second step, as I see it, is the claim that (8) is exceedingly improbable with respect to the amount and variety of evil we actually find; it seems very unlikely, he thinks, that there should be an omnipotent being so related to the evils that actually occur, that for each of them there is some good he could not have brought about without permitting that evil.

But here I doubt that Rowe has said quite what he meant. (8) isn't nearly so strenuous as it looks at first sight; it is satisfied, for example, if for each evil state of affairs that obtains there is a good state of affairs that includes it.[10] And this condition is satisfied if α, the actual world, is a good state of affairs – if α contains, on balance, more good than evil. We can see this as follows. If α is a good state of affairs, then for each evil state of affairs E that obtains, there is a good state of affairs – α – that includes it. But if a good state of affairs G includes an evil state of affairs E, then the conjunctive state of affairs G *and* E is equivalent to G (just as a proposition A is equivalent to $A \,\&\, B$ if A entails B.) If S and S^*, furthermore, are equivalent states of affairs, then it isn't possible that S be more valuable or less valuable than S^*; equivalent states of affairs will be equivalent in value. So for each evil state of affairs E that obtains, $E \,\&\, \alpha$ has the same value as α and hence is a good state of affairs; α, therefore outweighs B. Thus if α is a good state of affairs, the first condition of (8) is satisfied: for each evil E there is a good G – namely α – that outweighs it. But the second condition is also satsfied; even though God is omnipotent he could hardly have brought about α without permitting the evil states of affairs α includes. (8), therefore, is true if α is a good state of affairs.

Furthermore, it is plausible to think that (8) is true *only if* α is a good state of affairs. Suppose α is not a good state of affairs, and let E! be the largest evil state of affairs α includes. That is, E! is an evil state of affairs, is included by α, and includes every evil state of affairs α includes. E! is not, presumably, outweighed by any good state of affairs; if it were, α itself presumably, would be a good state of affairs. So if α is not a good state of affairs, the first condition of (8) is not satisfied.

Hence (8) is entailed by

(9) α is a good state of affairs

and is plausibly thought to entail it. So (8) is improbable with respect to E only if (9) is. I gather, however, that Rowe is not at all prepared to hold that (9) is particularly unlikely, either with respect to E or with respect to total evidence. So I don't think (8) is what is at issue here.

But then what *is* at issue? Perhaps

(10) Each evil E is such that God could not have actualized[11] a world as good as the actual world without permitting E.

This too won't be quite right; perhaps some evil E is such that God could have actualized a world as good as α without permitting E, but only by permitting some other evil or by deleting some good in such a way that the resulting possible world would have been no better than α. If so, God's goodness certainly wouldn't prevent him from permitting E. What really *is* at issue, here, I think, is the question whether God could have actualized a world that was just as good as the actual world, but contained less evil, overall. That is, I think Rowe means to say that the theist is committed to something like

(11) God is omnipotent, omniscient and wholly good; and every world God could have actualized that contains less evil than the actual world, contains a less favorable overall balance of good and evil than does the actual world;

or, equivalently,

(12) God is omnipotent, omniscient and wholly good; and the actual world is a better world than any world God could have actualized that contains less evil than the actual world.

Rowe means to hold, I think, that the theist is committed to (12) (he is obviously committed to its first conjunct; and perhaps Rowe believes that the first conjunct entails the second), and (12), he thinks, is improbable with respect to E.

Suppose this is what Rowe thinks: is he right? Perhaps not quite: the first conjunct of (12) by no means obviously entails the second. To approach this question, we should note first of all that for all we know, there is no such thing as *the* best world God could have actualized, or even *a* best world He could have actualized. Perhaps for every world W God could have actualized, there is a better world W^* he could have actualized. Of course we don't know that this is so; but we also don't know that it isn't so; and hence we can't properly assume that there is a best world God could have actualized. Now let's suppose that the total amount of *good*, – past, present and future – the actual world contains is 10^{18} felics. (Christianity suggests that the quantity of good α contains is in some sense infinite, for it suggests that there is a future time after which there will forever be continuing and ever more impressive good. If this were so, our term '10^{18} felics' would be misleading in that it suggests a finite quantity of good; but let's let that pass.) The actual world, therefore, contains 10^{18} felics. Now it is possible that every world God could have actualized and which contains 10^{18} felics of good, also contains some evil. And it could be that among the worlds God could have actualized that contain 10^{18} felics of good, there is no world W that contains a least amount of evil. Just as there might be no best world God could have actualized, so there might be no best world containing 10^{18} felics God could have actualized; it could be that for every world God could have actualized containing 10^{18} felics of good and r units of evil, there is another world he could have actualized containing 10^{18} felics of good and less evil. Consider the class of worlds C such that W is a member of C if and only if (1) God could have actualized W, and (2) W contains 10^{18} felics of good. One possiblity for all we know, is as follows. First, there are uncountably many worlds in C. Second, every world in C contains at least ℓ turps of evil (where ℓ is some number less than but very close to $10^{13} - 10^{12.999\ldots9}$, let's say), but no world in C contains exactly ℓ turps. And finally, for any real number ϵ, there is a member of C such that the amount of evil it contains is within ϵ of ℓ. If this were so, then among the worlds God could have actualized that contain 10^{18} felics of good, there wouldn't be any containing a least amount of evil.

But if this were so the second conjunct of (12) wouldn't be true; *no* world that contained 10^{18} felics and was such that God could have actualized it, would then be better than every world God could have actualized that contained less evil. In no way, of course, would this compromise God's goodness; if there is no best world He could have actualized, He can hardly be faulted for failing to actualize the best world He could have actualized. The most that could sensibly be required is that the evil in the actual world be *near* ℓ – where nearness can be spelled out in any way, short of identity, the atheologian pleases.

But suppose we waive this consideration and concede, for purposes of argument, that the theist is after all committed to (12); let us suppose that G entails (12). There's no reason whatever to think this is true; but let's suppose so anyway. Rowe's second step is to claim that (12) is improbable, given the amount and variety of evil we do in fact find; that is, (12) is improbable with respect to E. And if (12) is entailed by G, then G is improbable with respect to E.

But *is* (12) improbable with respect to E? Why should we think so? Are there grounds for this claim? How are we supposed to tell whether or not (12) is improbable with respect to E? Do we just take a good look at these two propositions? What considerations are relevant to determining whether Rowe's claim is true? How can we proceed here?

III. EVIL AND 'A PRIORI' PROBABILITIES

Perhaps we can make progress by inquiring into the relationship which holds between A and B when B is evidence for A or when A is probable with respect to B. Just what relation is it that the atheologian says holds between E and G when he says the former is evidence against the latter, or that $\overline{G}$, the denial of G, is probable with respect to E? The nature of this relationship is, of course, subject to enormous dispute; this area of inquiry bristles with paradox, mystery, and confusion; it is fraught with difficulty, darkness, and despair. Where can we start? Perhaps with the calculus of probabilities. Although this area is indeed racked by disagreement, most[12] workers in it take it that the relationship between a pair of propositions A and B when B is evidence for or against A is a relationship that conforms to the calculus of probabilities, or is based on a relationship that so conforms. Presumably, then, the atheologian will hold that the relationship he characteristically

ascribes to G and E conforms to this calculus. (At any rate if he doesn't suppose this, it is hard to see how even to consider his claim without further explanation on his part.)

We shall take probability to be a three termed relation among a pair of propositions and a real number. Where A, B and C[13] are variables ranging over propositions and r a variable ranging over real numbers, $P(A/B) = r$ is the proposition that the probability of A with respect to B is r; $\bar{A}$ is the denial or negation of A; $A \,\&\, B$ is the conjunction of A with B; and $A \vee B$ is the disjunction of A with B. Axioms for the calculus of probabilities may then be stated as follows.[14]

A_1 $\quad P(A/B) = r, 0 \leqslant r \leqslant 1.$

A_2 $\quad$ If B entails A, $P(A/B) = 1.$

A_3 $\quad$ If $A \,\&\, B \,\&\, C$ is not possible and A is possible, $P((B \vee C)/A) = P(B/A) \times P(C/A).$

A_4 $\quad P((A \,\&\, B)/C) = P(A/C) \times P(B/(A \,\&\, C)).$

A_5 $\quad$ If A entails B and B entails A, then $P(A/C) = P(B/C)$ and $P(C/A) = P(C/B).$

A couple of explanatory remarks about these axioms.[15] A_1 says simply that for any two propositions A and B, the probability of A on B is between 0 and 1 (including those endpoints). It therefore asserts that the probability relation is *connected* among propositions; any pair of propositions are thus such that the first has some probability or other on the second. According to A_2, if B entails A, then A has maximal probability with respect to B; the logical consequences of a proposition have maximal probability with respect to it. A_3, the *Addition* Axiom, says that, given the conditions stipulated in its antecedent, the probability of a disjunction on given evidence is equal to the sum of the probabilities of the disjuncts on that evidence. A_4 is the *Multiplicative* Axiom; it states that the probability of a conjunction, on given evidence, is equal to the probability of the first conjunct, on that evidence, multiplied by that of the second on the conjunction of the first with the given evidence. A_5, finally, says simply that logically equivalent propositions have the same probability on given evidence, and that a given proposition has the same probability on logically equivalent propositions. Thus logically equivalent propositions are intersubstitutible in probability contexts *salva veritate*.

There are a few theorems that will prove useful in our subsequent discussion: I shall state them without proof.

T_1 If B is possible, $P(A/B) = 1 - P(\bar{A}/B)$

T_2 If $P(B/C) \neq 0$, then $P(A/(B \,\&\, C)) = \dfrac{P((A \,\&\, B)/C)}{P(B/C)}$

T_3 If $P(B/C) \neq 0$, then $P(A/(B \,\&\, C)) = \dfrac{P(A/C) \times P(B/(A \,\&\, C))}{P(B/C)}$

T_3 is called 'Bayes' Theorem'; it is named for the distinguished 18th century clergyman Thomas Bayes. (It is not recorded whether Bayes found his Theorem useful in pastoral work.) T_3 is sometimes called 'the Inverse Probability Theorem'; given background evidence C, T_3 gives us the probability of A on B in terms of the probability of B on A (together with the probabilities of A and B on that information).

T_4 If $P(A/B) = n$ and A entails C, then $P(C/B) \geqslant n$.

That is, the logical consequences of a proposition are at least as probable, on given evidence, as is the proposition itself. If we say that A *confirms* B when $P(B/A) > 1/2$[16], then we may say that a proposition confirms the logical consequences of any proposition it confirms.

Finally, a corollary of T_4 is

T_5 If $P(A/B) = n$ and C entails A, then $P(C/A) \leqslant n$.

That is, what entails a proposition A is no more probable, on given evidence, than A. Hence if a proposition A *disconfirms* B (if, that is, $P(B/A) < 1/2$), then A disconfirms every proposition that entails B.

Thus lightly armed, let us return to our subject. The atheologian's claim, you recall, is that

G God is omnipotent, omniscient and wholly good

is improbable on

E There are 10^{13} turps of evil.

Rowe argued that G is improbable on E because G entails

(12) God is omnipotent, omniscient and wholly good; and the actual world is a better world than any world God could have actualized that contains less evil;

but (12), he says, is improbable on E; hence G is improbable on E. We noted that G doesn't at all obviously entail E; but in any event the atheologian is committed to supposing that (12) is improbable on E if he means to hold that G is. (12) entails G; hence (by T_5) G is improbable on E only if (12) is.

A central atheological claim, then, is that $P((12)/E) \leqslant 1/2$. As I have already remarked, this is neither self-evident, or obvious, or generally accepted by all parties to the dispute, or part of the current lore about probability, or accepted by all rational persons or all the relevant experts. Why then should we think it true? How could we tell? What would be relevant to deciding? Perhaps the following will provide a bit more insight.

First of all, let's revise (12) a bit to

(12*) God is omnipotent, omniscient and wholly good and God could not have actualized a world as good as the actual world but containing less evil than the 10^{13} turps of evil the actual world contains.

The atheologian, of course, is committed to holding that (12*), like (12), is improbable on E; for (12*), like (12), entails G; and he holds that G is improbable on E. Now (12*) entails that God has actualized the actual world (i.e., whatever world is in fact actual); its first conjunct entails that God exists, and it is a necessary truth that if God exists, then he has actualized whatever world is, in fact, actual. (If you don't think this is a necessary truth, just revise (12*) so that it reads 'God is the omnipotent, omniscient and wholly good creator of the world; and...'). Now let T be any tautology – *either some ants are parasitic or none are*, for example. According to Bayes' Theorem,

$$(13) \qquad P((12^*)/(E \,\&\, T)) = \frac{P(12^*/T) \times P(E/(12^* \,\&\, T))}{P(E/T)}.$$

Since any proposition is logically equivalent to its conjunction with a tautology, E & T is equivalent to E and (12) & T to (12). By A_5, therefore, (13) is equivalent to

$$(14) \qquad P((12^*)/E) = \frac{P((12^*)/T) \times P(E/(12^*))}{P(E/T)}.$$

(12*), furthermore, entails E; for if the actual world contains 10^{13} turps of evil, then E is true. Hence, by A_2, $P(E/(12^*)) = 1$; so (14) is equivalent to

$$(15) \qquad P((12^*)/\mathrm{E}) = \frac{P((12^*)/T)}{P(\mathrm{E}/T)}.$$

Let's say that the probability of a proposition with respect to a tautology is its *a priori* probability. What follows, then, is that $P((12^*)/\mathrm{E})$ is equal to the *a priori* probability of (12^*) divided by the *a priori* probability of E. Hence the atheological claim – that $P(\mathrm{G}/\mathrm{E}) < 1/2$ – is true only if the *a priori* probability of (12^*) is less than 1/2 the *a priori* probability of E. That is, E disconfirms G only if the probability of E with respect to a tautology is more than twice that of (12^*) with respect to that tautology. Suppose *per impossible*, all you knew were some tautologies. If the atheologian is right about E and (12^*), E would have to be more than twice as probable, on what you know, as (12^*).

We may approach the same point from a slightly different direction. By the multiplicative axiom,

$$(16) \qquad P((\mathrm{E} \,\&\, \mathrm{G})/T) = P(\mathrm{E}/\mathrm{T}) \times P(\mathrm{G}/(\mathrm{E} \,\&\, \mathrm{T})).$$

Hence

$$(17) \qquad P(\mathrm{G}/(\mathrm{E} \,\&\, T)) = \frac{P((\mathrm{E} \,\&\, \mathrm{G})/T)}{P(\mathrm{E}/T)}.$$

and by A_5

$$(18) \qquad P(\mathrm{G}/\mathrm{E}) = \frac{P((\mathrm{G} \,\&\, \mathrm{E})/T)}{P(\mathrm{E}/T)}$$

If the atheological claim is correct, therefore, the *a priori* probability of G & E must be less than half that of E.

But how could the probability of E with respect to a tautology be twice as great as that of G & E or (12^*)? How could a tautology discriminate thus among propositions? Indeed, does it make sense to say that E or G & E or (12^*) *have* a probability on a tautology? How could a contingent proposition have a probability on a tautology? What sense can we make of these *a priori* probabilities? To answer these questions, we must inquire more closely into the nature of the relationship represented by '$P(A/B) = n$'. That is to say, we shall have to consider how the probability calculus is to be interpreted.

There are nowadays three important interpretations of the probability calculus (with several sub-varieties of each): the *logical* interpretation, ac-

cording to which the probability relation is a quasi-logical relation of which entailment is a special case, the *frequency* interpretation which holds that probability statements essentially record the relative frequency of events of one sort among events of another, and the *personalist* interpretation, according to which basic probability statements essentially record the degree to which someone accepts some proposition or other. What I propose to do is to see what this *a priori* probability might be and what the atheologian's argument looks like from the perspective of each of these interpretations.

IV. PERSONALISM AND ATHEOLOGY

I turn first to the personalist interpretation of the Probability Calculus, which goes back to F. P. Ramsey and has been espoused, in one form or another, by Bruno de Finetti,[17] Richard Jeffrey,[18] and Ian Hacking.[19] The personalist classically holds that for each person S there is a *credence function*: a function from the set of propositions believed by S into the real numbers between 0 and 1. $\mathrm{Ps}(A) = n$, then, records something like the degree to which S believes or accepts A; $\mathrm{Ps}(A) = 1$ proclaims S's utter and absolute endorsement of A while $\mathrm{Ps}(A) = 0$ records S's entire and whole-hearted commitment to $\bar{A}$. $\mathrm{Ps}(A)$ measures S's *partial belief* that A. For most persons S, for example, $\mathrm{Ps}(\textit{Tucson is in Arizona}) > \mathrm{Ps}(\textit{London, England is north of Saskatoon, Saskatchewan})$. Personalists sometimes claim to be able to measure S's partial belief that A by engaging him in certain betting situations involving A. (A bit of skepticism seems justified when A is something like *God has created the world*; and in any event there is no guarantee that S will be willing to take part in any of these wagers.)

$\mathrm{Ps}(A/B)$, on the other hand, measures S's *conditional* partial belief – his partial belief that A, given the condition that B. $\mathrm{Ps}(A/B)$ is defined thus:

$$\mathrm{Ps}(A/B) = \frac{\mathrm{Ps}(A \,\&\, B)}{\mathrm{Ps}(B)}, \quad \text{provided that } \mathrm{Ps}(B) \neq 0.$$

(Compare this expression with A_4 above.) It was the *a priori* probabilities of G and E – their probabilities on tautological evidence – that gave us pause before; but from the present point of view *a priori* probabilities, like Caesar's wife Calpurnia, are entirely above reproach. Let's suppose S's credence function Ps conforms to the calculus of probabilities. This does not guarantee that there is some proposition $A - B$ *or* $\bar{B}$, for example – such that $\mathrm{Ps}(A) =$

1. However, we can easily enough *normalize* S's credence function by multiplying $\mathrm{Ps}(A)$, for every A, by $1/n$, where n is the highest value Ps assigns. Then $\mathrm{Ps}(T) = 1$, where T is any tautology; and $\mathrm{Ps}(A/T) = \mathrm{Ps}(A \,\&\, T)/\mathrm{Ps}(T) = \mathrm{Ps}(A)/1 = \mathrm{Ps}(A)$: S's partial belief that A. So if Ps conforms to the calculus of probabilities, the *a priori* probability of a proposition (with respect to S) is simply the degree to which S believes that proposition. Here there is nothing to give us pause; the notion of *a priori* probability, from this point of view, occasions no special difficulty, $\mathrm{Ps}(\mathrm{G}/T) < 1/2\ \mathrm{Ps}(\mathrm{E}/T)$, is true if and only if S believes E more than twice as firmly as he believes G. And while the notion of a quantitative measure of belief may conceal substantial difficulty, the general idea is easy enough to grasp and presents no particular perplexities.

But if it presents no overwhelming perplexities, it is also hard to see how the atheologian can make use of the personalist interpretation in presenting a probabilistic argument for atheism, or a probabilistic objection to theism. Indeed, the view as so far presented has no *normative* aspect at all; hence it can hardly be used to show that the theist's beliefs are noetically irresponsible or lacking in intellectual respectability. The personalist typically adds, however, that a person's beliefs are *rational* only if they are *coherent* – only if, that is, they conform to the probability calculus. Thus my beliefs are incoherent if, say, $\mathrm{Pp}(A) = 0.7$, $\mathrm{Pp}(B) = 0.5$ and A entails B. This requirement doesn't look unduly stringent, and, as we shall see in a moment, from a certain point of view it is permissive indeed.

But from another point of view it is hopelessly strong. In the first place, if Ps conforms to the probability calculus, then $\mathrm{Ps}(\mathrm{T}) = 1$ for every necessary truth T – even those whose truth isn't presently known (as well as those S has not so far had the pleasure of hearing of). Since either Goldbach's conjecture or its denial is a necessary truth, S, if he is coherent, will embrace Goldbach's conjecture with maximum enthusiasm or else thus embrace its denial – and this despite the fact that no one knows whether or not Goldbach's conjecture is true. Similarly A_2 requires that if S believes A, then S must believe the logical consequences of A at least as strongly as he believes A. This, again, is a bit too much to expect, since, unless S is exceptionally well informed, he will not have heard of nearly all the consequences of what he believes. We could perhaps make a stab at ameliorating this difficulty by restricting the range of the implicit universal quantifiers in $\mathrm{A}_1 - \mathrm{A}_5$ to the set of propositions accepted (to some degree or other) by S. Even if we do so, however, coherence still requires that S believe every necessary truth

he believes to the same degree; that is, Ps(N) = 1 for any necessary truth N that is in the domain of *S*'s credence function. This might be thought to set an admirably high standard; but the fact is a rational person will not, typically, believe all the necessary truths he believes, with equal enthusiasm. He will not, for example, believe Gödel's Second Theorem (or the truth that there are no nonexistent objects or that what is possible does not vary from world to world) to the same degree that he believes $2 + 1 = 3$ or the corresponding conditional of Modus Ponens.

But suppose we waive all this and concede, for purposes of argument, that if *S* is rational, then his credence function Ps is coherent. How are we to construe, from the personalist point of view, the atheological claim that $P(G/E) < 1/2$? As a truth about the *atheologian*, i.e., as $Pa(G \& E)/Pa(E) < 1/2$? While this information may possess a certain biographical charm, it is certainly not unexpected; we already knew that the atheologian's degree of belief in G (and hence in G & E) is low. And what is there here to disturb the theist? This bit of news about the atheologian's credence function is perhaps moderately interesting (at least to his friends and relatives) but how does it constitute an objection to theistic belief? How is this an objection to theistic belief? It is hard indeed to see here any way at all to construct an argument for atheism or for the conclusion that theism is untenable.[20]

Perhaps the personalist atheologian is to be construed as claiming that $P(G/E) < 1/2$ is *rational*, that is, can be embedded in a coherent set of beliefs. This is certainly true, but of dubious relevance. For the theist can retort that $P(G/E) > 1/2$ or even $P(G/E) = 1$ are rational in this sense; they too can be embodied in a coherent set of beliefs. Indeed, if G and E are logically independent, then for any real number r, $P(G/E) = r$ can be embedded in a set of beliefs that conforms to the probability calculus.

Accordingly, the personalist interpretation offers no aid at all to the atheologian intent on a probabilistic argument from evil. On the official personalist view, coherence is necessary and sufficient for rational acceptability. So any set of beliefs is rational if coherent; and any consistent belief is rational in that it can be embedded in a coherent set of beliefs. This is indeed a counsel of tolerance; it tolerates sheer absurdity. From this point of view, I could be perfectly rational in believing that the sun will not rise tomorrow, even if my evidence on this topic is just like everyone else's. It is therefore no wonder that personalism doesn't offer much to the atheologian intent on convicting the theist of holding untenable beliefs; and this may

plausibly be seen not so much as a point in favor of theism as a point against personalism.[21]

V. ATHEOLOGY AND THE LOGICAL CONCEPT OF PROBABILITY

I turn, therefore, to the second of the three interpretations of the probability calculus: the logical interpretation developed by Carnap[22] and his followers. According to the official personalist view, as we have seen, a necessary (and perhaps sufficient) condition of S's beliefs being rational is that they be coherent, conform to the probability calculus. Now the heart and soul of the logical interpretation is that for any propositions A and B, A has a certain perfectly objective probability with respect to B. From this point of view, therefore, the logical interpretation may be seen as laying down further necessary conditions of rationality; it specifies that just one of the many coherent credence functions is in fact correct. A statement of the form $P(A/B) = n$, therefore may be thought of as specifying the rational degree of belief that A, given B as evidence. Perhaps the rational degree of belief in

(19) Feike can swim

given

(20) Nine out of ten Frisians can swim and Feike is a Frisian

is 0.9; that is,

(21) P((19)/(20)) = 0.9.

If so, the rational person who knows (20) and knows nothing else relevant will believe (19) to degree 0.9.

More fundamentally, however, on the logical interpretation such a proposition as (21) is not first of all about what is or isn't rational; fundamentally, (21) records an objective, quasi-logical fact about (19) and (20). It's truth in no way depends upon anyone's belief or knowledge, or upon any other contingent state of affairs. From this point of view, the probability relation may be thought of as *partial entailment*, with entailment *simpliciter* the limiting case where $P(A/B) = 1$. (20) does not entail (19); but perhaps we may say that the former *partially* entails the latter; this reflects the strictly logical character of probability. And just as *A entails B* is *necessarily* true if true at all, so propositions like (21) are either necessarily true or necessarily

false. Probability thus construed has nothing to do with partial belief, or uncertainty, or lack of knowledge; God may not *need* probability, but he nonetheless knows the value of $P(A/B)$ for any propositions A and B. So if our atheologian accepts the logical interpretation of probability, he will hold that

(22) $P(\mathrm{G/E}) < 1/2$

and

(23) $P((12^*)/\mathrm{E}) < 1/2$

state necessary truths about G, (12^*), and E – truths in no way relative to anyone's knowledge or belief.

A. What about 'A Priori' Probabilities?

As we have seen, the probabilistic atheologian is committed to holding that $P((12^*)/T) < 1/2\ P(\mathrm{E}/T)$, where T is any tautology. We have been calling $P((12^*)/T)$ and $P(\mathrm{G}/T)$ the *a priori* probabilities of (12^*) and G. From the point of view of the logical interpretation, however, this is something of a misnomer; it certainly doesn't seem possible to discover *a priori* what $P(\mathrm{G}/T)$ or $P(\mathrm{E}/T)$ might be. (Maybe it's no easier *a posteriori*; but it certainly can't be done *a priori*). Of course the central thrust of the logical view is no such epistemological claim as that these probabilities can be discovered *a priori*, but just that every proposition has a definite, wholly objective probability on a tautology.

But is there really any reason at all to think this is *true*? How could a tautology –

(24) Either some ants are parasitic or none are

– for example, be probabilistically relevant to such contingent propositions as, for example, G or E or

(25) Paul L. Zwier is wearing an orange shirt?

And even if tautologies are thus probabilistically relevant, shouldn't all contingent propositions have the *same* probability with respect to a tautology? (24), I should think, gives us no reason to accept (25) and none to accept its denial. If we know only tautologies, it would seem that we have just

as much reason to think (25) true as to think it false: in each case we have no reason at all. Consider E or G or (12*); if all I knew were tautologies, shouldn't I take these propositions to be equi-probable with their denials?

The answer, of course, is *no*. Let A be any contingent proposition and B any proposition logically independent of A. A is equivalent to $(A \,\&\, B) \vee (A \,\&\, \bar{B})$. If every contingent proposition has an *a priori* probability of 1/2, $P(A \,\&\, B)/\mathrm{T}) = 1/2$ and $P((A \,\&\, \bar{B})/T) = 1/2$. But then by A_3 $P((A \,\&\, B) \vee (A \,\&\, \bar{B}))/T) = 1$. $P(A/T)$, however is 1/2, since A is contingent. Hence $P(A/T) = 1/2$ while $P((A \,\&\, B) \vee (A \,\&\, \bar{B}))/T) = 1$; and this violates A_5.

We might retort that it is *simple* or *atomic* contingent propositions that have an *a priori* probability of 1/2. But this too is unsatisfactory. The deeper reason is that the notions of simplicity and atomicity, while perfectly at home in formal systems, where one deals with *sentences*, have no clear application to propositions. This is a large and difficult topic, however, and here we need not enter it. For suppose the notion of atomicity did apply to propositions: what would be the *a priori* probability of *there are just 17 horses*? Propositions of this sort clearly cannot have an *a priori* probability of 1/2; if they did,

(26) There are either 17, 18 or 19 horses

would have (by A_3) an *a priori* probability of $\frac{3}{2}$, which is frowned upon in these contexts.

You might complain that *there are just 17 horses* is not an atomic or simple proposition, but has the quantificational complexity its symbolization in first-order logic suggests. Very well, consider a small spot and name it 'Sam'. If *Sam is red, Sam is blue* and *Sam is yellow* each have an *a priori* probability of 1/2, we have (by A_3) the same unfortunate circumstance as with (26).

Contingent propositions thus cannot all have *a priori* probability of 1/2; but then they can't all have the *same* probability. For if they don't all have an *a priori* probability of 1/2, then some must have an *a priori* probability different from that of their contingent denials. Nor is it plausible to think that every atomic proposition has the same *a priori* probability (if indeed, a distinction between atomic and complex propositions can be sustained). For return to the spot we named 'Sam' and consider

(27) Sam is colored.

Will (27) be *simple* or *complex*? We might be tempted to hold that it is complex on the grounds that it is not logically independent of such simple

propositions as *Sam is red, Sam is blue*, and the like. But if we require that simple propositions be logically independent of each other, then the above pair won't qualify as simple – despite the fact that they seem to be as good candidates for simplicity as any we can think of. We might try arguing instead that (27) is complex because it is equivalent to a vast disjunction that contains a different disjunct for each color. But do we have any reason to think that there is a determinate number of colors? And even if there is, then *Sam is red* can be seen to be equivalent to a conjunction which contains, for each color *C* except red, a conjunct of the form *Sam is not C*. It is thus exceedingly difficult to find grounds for holding that (27) is complex. But if it is simple, then not all simple contingent propositions will have the same *a priori* probability; for surely (27) and *Sam is red* will not have the same *a priori* probability. Accordingly, contingent propositions and indeed atomic contingent propositions do not all have the same *a priori* probability.

But how can a tautology thus discriminate among contingent propositions, ascribing to some a higher probability than it ascribes to others? Is there, in fact, any reason to suppose that contingent propositions *have a priori* probabilities? The question is not, of course, whether perhaps their *a priori* probability is zero; the question is whether they have any degree of probability at all, in the logical sense, with respect to tautologies. To ask for the probability of G or E on a tautology might be like asking for the temperature of the number nine. The temperature of nine isn't zero (F, C or K); it *has* no temperature. Similarly, perhaps G and E have no *a priori* probability. Perhaps the probability relation (the relation in which A and B stand when there is a number n such that $P(A/B) = n$) isn't *connected* among propositions; perhaps it doesn't hold between A and B when A is contingent and B a tautology. It is easy to see why the proposition *Feike can swim* would be thought to have a probability with respect to *9 out of 10 Frisians can swim and Feike is a Frisian*; but does it have a probability with respect to *Either Feike is a Frisian or he isn't*? What actually or allegedly determines the *a priori* probability of a proposition? What, if anything, is it supposed to depend upon?

B. Simplicity

According to Richard Swinburne, the answer is *simplicity*; the simpler a hypothesis is, the greater its *a priori* probability.

...where we ascribe greater intrinsic [i.e., *a priori*] probability to one of two hypotheses of equal content, intrinsic probability seems to be a matter of simplicity, in that wide sense of the term in which it is used by philosophers of science.... The simpler an hypothesis *is* ... the greater its intrinsic probability.[23]

Any number of hypotheses are ordinarily supported by our evidence; the one we choose is the one with greatest probability on that evidence, which it typically has by virtue of greatest *a priori* probability, which it has by virtue of greatest simplicity. The problem of curve-fitting provides an example. You have a lot of data; you plot it on a graph; and then of all the lines that can be drawn through the points you have plotted, you choose the simplest and smoothest. You choose the simplest function (the one represented by the simplest and smoothest curve) even though as many functions as you please are in fact supported by your evidence. And in so doing you are perfectly justified, says Swinburne, because the simpler a hypothesis is the greater its *a priori* probability.

But this suggestion cannot be right. In the first place, the notion of simplicity as a property of *propositions* as opposed to *sentences* is at best problematic. Although this is a deep and important difficulty, discussing it here would take us too far afield. But even if we waive this problem we can see that *a priori* probability, if there is such a thing, does not depend in any straight-forward way upon simplicity. In particular, it is not true that the simpler a proposition, the greater its *a priori* probability. For if that *were* true, all logically equivalent propositions would be equally simple (since they have the same *a priori* probability) and no contingent proposition could be simpler than any necessary proposition (since every necessary proposition has an *a priori* probability of 1). But consider the following propositions:

(28) There are nine planets.

(29) There are $\int_0^3 x^2\,dx$ planets; and if, for every set S and for every function f from S onto the power set of S, there exists the subset S^* of S such that x is a member of S^* if and only if x is not a member of $f(x)$, then there is no universal set.

(28) and (29) are equivalent – that is, true in the very same possible worlds; but one could hardly claim, with a straight face anyway, that they are equally simple. The second conjunct of (29), furthermore, certainly looks a lot less simple that (28); but it is necessary and (28) is contingent. The second conjunct of (29) is also equivalent to

(30) Either Feike is a Frisian or it is false that Feike is a Frisian;

but it certainly seems less simple.

Note also that

(31) If A entails B, then the *a priori* probability of B equals or exceeds that of A;

is provable in the probability calculus; and consider such propositions as

(32) There are no horses

and

(33) There are fewer than 10^{17} horses.

Surely (32) is simpler than (33); so on the suggestion in question, (32) should have greater *a priori* probability than (33). But (32) entails (33) and hence (by 31) cannot have greater *a priori* probability. Accordingly, *a priori* probability does not depend in this way upon simplicity.

C. Content

A suggestion endorsed by Carnap and others is that it is *content* that determines *a priori* probability; the greater content a proposition has, the smaller its *a priori* probability. The content of a proposition is a matter of how much it *says*; and the more it says, the more likely it is to go wrong. On this suggestion necessary truths have no content at all (and hence have maximal probability on tautological evidence); and of course all necessary propositions have the same degree of content. This is initially anomalous; is it really correct to say that Gödel's Incompleteness Theorem, for example, has no more content than 2 + 1 = 3 on the grounds that both are necessary? But suppose we waive this objection for the moment.

If A entails but is not entailed by B, then, on the present view, A has more content (and hence less *a priori* probability) than B. Consider, for example,

(34) There are just 2 horses present

and

(35) There are some horses present.

Here it seems initially plausible to say that (34) has more content than (35);

(34) says everything that (35) says and more: it says just how many horses are present. But things begin to look less inviting when we compare (35) with, say, the Fundamental Theorem of the Calculus. The latter is necessary, and hence entailed by every proposition, including (35); but is it really sensible to say that (35) says everything the Fundamental Theorem says and more? This notion of content clearly needs a lot of explanation; but insofar as we do have a rough and ready grasp of the idea of propositional content we surely can't say, with any show of plausibility, that (35) has more content than the Fundamental Theorem of the Calculus. And surely it is simply wrong to say that all necessary propositions – Gödel's Theorem, for example – have no content; many of them, clearly enough, have a great deal of content. *What will be, will be*, perhaps, has no content; and if I assert it, perhaps I merit the scorn that properly attaches to those given to empty and contentless assertion. But surely Gödel's Theorem and any number of similar necessary truths are not thus contentless. And we can't object, here, that this notion of content is a *technical* one, to be explained in terms of something else (*a priori* probability, for example); for in the present context we are using this notion to explain *a priori* probability. But suppose we waive this objection also.

There are still more serious difficulties. It makes a certain rough sense to say that (34) has more content than (35); (34) entails but is not entailed by (35). But what about logically independent propositions? Can we compare their content? Is there a reason to think they are comparable with respect to content? Does

(36) There is at least one horse

have more content than

(37) There is at least one mouse

on the grounds, perhaps, that horses are larger than mice? Or do they have the same content? What about (36) and

(38) There are just 3 mice?

Or (36) and

(39) There are at least 8022 mice

(given that the weight of the average horse is equal to that of 8022 mice, to the nearest mouse)? Is there any reason to think that (36) and (37), (38) or (39) are comparable with respect to content?

And suppose a proposition A entails a proposition B, so that A can plausibly be said to have more content than B; what could it mean to say that A has *at least twice as much content* as B? An atheologian who accepts the Carnapian interpretation of probability will hold that (12^*) has more than twice the content of E (since he holds that $P(E/T)$ is more than twice as great as $P((12^*)/T)$. But even granted that we can make sense of this suggestion, what earthly reason is there for thinking it *true*? Why couldn't the theist sensibly claim that as *he* sees things, E has about 9/10 the content of (12^*), so that $P((12^*)/E)$ (and hence $P(G/E)$) is about 0.9? Then the existence of God would be highly probable on that of evil. This notion of content is slippery indeed. Suppose these propositions are comparable in content; how could one possibly tell what ratios their contents stand in?

D. Deeper Troubles

There is a more profound difficulty with the logical interpretation. It isn't just that there is no reason to think a contingent proposition has an *a priori* probability, nor even that if it did, there seems to be no way at all of telling what it might be. The fact is there seems to be no way to assign content and hence *a priori* probability in a way that is consistent both with the probability calculus and with intuition. Consider

(40) There are some horses

(41) There are between 2 and 2000 horses

and

(42) There are exactly 3 horses.

It seems plausible to say that (40) has less content than (41), which has less content than (42); hence it may be sensible to say, so far, that $P((40)/T) > P((41)/T) > P((42)/T)$. Now such propositions as (42) are maximally specific with respect to specifying how many horses there are; they should therefore have the same content and, on the present suggestion, the same *a priori* probability. Consider, therefore, the following set H:

$$H \begin{cases} H_0 & \text{There are no horses} \\ H_1 & \text{There is just 1 horse} \\ H_2 & \text{There are just 2 horses} \\ \vdots & \\ H_n & \text{There are just } n \text{ horses} \\ \vdots & \end{cases}$$

H is a countably infinite set of propositions; for each natural number n, H contains a member according to which there are just n horses. All the members of H are maximally specific with respect to the question how many horses there are; presumably, therefore, all have the same content. If so, they will have the same *a priori* probability. But the members of H can have the *same a priori* probability only if they all have *a priori* probability *zero*. For if each H_i has the same non-zero *a priori* probability, then there will be a finite subset H^* of H such that

(43) $H_0 \vee H_1 \vee H_2 \vee \ldots \vee H_n$,

the disjunction of the members of H^*, will have (by repeated applications of A_3) an *a priori* probability greater than 1. The members of H must therefore have zero *a priori* probability. This, however, leads to some paradoxical results.

Consider

(44) If $C = \{A_0, A_1 \ldots A_n\}$ is a finite set of propositions that are mutually exclusive in pairs and jointly exhaustive, then $\sum_0^n P(Ai/T) = 1$.

(44) says that the *a priori* probabilities of any finite set of propositions meeting the conditions laid down in its antecedent sum to 1; this is provable in the probability calculus. A natural generalization of (44) is

(45) If $C = \{A_0, A_1 \ldots A_n \ldots\}$ is a countable set of propositions that are mutually exclusive in pairs and jointly exhaustive, then $\sum_0^\infty P(Ai/T) = 1$.

(45) is sometimes called 'Countable Additivity'. Although it is not provable in the probability calculus, (45) has great intuitive appeal: to return to our

set *H*, the proposition that some member of it is true, has an *a priori* probability of 1^{24}; if so, shouldn't the *a priori* probabilities of its members sum to 1? If we take the members of H to have equal content and equal *a priori* probability, however, we can't accept (45); as we have seen, the only way to assign equal probability to the members of H is to assign them all zero probability. Of course we might find it only mildly paradoxical to reject (45); the notion of addition it embodies essentially involves the idea of a limit of an infinite series, and is thus quite different from the notion of addition embodied in A_3. Still, it seems at best peculiar to suppose that each member of *H* has zero probability while there is a probability of 1 that some member of *H* is true.

But perhaps this isn't so strange; one's intuitions get a severe bending in this area. Perhaps the right course *is* to insist that each member of *H* has zero *a priori* probability. It is hard, indeed, to see how things could go otherwise; clearly the members of *H* have the same amount of content (though possible worlds in which, say, there are just 10^8 horses will contain a lot more horseflesh than those with just 2 horses) and hence, on the present suggestion, they have the same *a priori* probability. And even apart from the idea that *a priori* probability depends upon content, it seems clear that if the members of *H* have any *a priori* probability at all, they have the same *a priori* probability; for surely no number is more likely on tautological evidence, to be the number of horses than any other. Accordingly, the *a priori* probability of the members of *H* must be zero.

This has some interesting consequences. In the first place, it follows that contingent universal propositions – propositions of the form

(46) All *A*'s are *B*

will have zero *a priori* probability, provided that for any number *n* it is logically possible that there be *n* *A*'s that are not *B*. For every universal proposition is equivalent to one of the sort we find in *H*;

(47) All crows are black,

for example, is equivalent to

(48) There are no non-black crows.

Since for any number *n* it is logically possible that there be *n* non-black crows, (48) has an *a priori* probability of zero; but then the same goes for

(47). Carnap manfully accepted this consequence and asserted that in fact the *a priori* probability of such universal propositions *is* zero. And the fact is, it seems exceedingly hard to avoid this conclusion; the argument for it (given the premiss that such propositions *have a priori* probability) is simple straightforward, and compelling.

Nevertheless, the consequences are thoroughly paradoxical. In the first place, if the *a priori* probability of a proposition is zero, its probability on any other proposition will also be zero: $P(A/B) = P(A \,\&\, B)/P(B)$; but $P(A \,\&\, B)$ will be zero if $P(A)$ is. This means that no amount of empirical evidence can confirm a universal generalization. No matter what our evidence, the probability of Newton's Laws or Special Relativity with respect to it is zero. Carnap and others[25] have been prepared to accept this consequence too, arguing that what we are really interested in is not the probability of the Universal Generalization (over all space and time) *All A's are B*, but rather some more restricted proposition: some version, perhaps, of *the next A will be a B*[26]. And perhaps this is not totally implausible.

There is another obvious consequence, however, that has not received nearly the attention it deserves: if every universal proposition has zero *a priori* probability, every existential proposition will have an *a priori* probability of 1. So, for example, the proposition

(49) There is at least one horse

will have an *a priori* probability of 1, and hence a probability of 1 on any evidence. Indeed, the *a priori* probability of

(50) There are more than 10^{10} horses

will also be 1; for any integer n you pick, the *a priori* probability of

(51) There are more than n horses

will be 1. (*Proof*: by repeated applications of A_3, the disjunction *there are 0 horses* v *there is just 1 horse* v ... v *there are just n horses* will have *a priori* probability of zero and hence its denial, i.e., (51), will have an *a priori* probability of 1.) Unless there are infinitely many horses, therefore, there will be some number n such that, while there are n horses, the proposition *there are n horses* has zero probability on any evidence.

And of course this conclusion does not hold only for horses; it holds for any sort of object such that it is possible for any natural number n, that there

be n objects of that sort. Hence the *a priori* probability that there are witches, Siberian Cheesehounds and Homeric gods is 1. Some versions of the Free Will Defense have sometimes been scored for making the allegedly improbable assumption that there are demons-malicious non-human free spirits. (Of course the fact is these versions don't presuppose that there are any such things, but only that there being some is consistent with G.) From the present point of view, however, this assumption is remarkably unventuresome if not laughably trivial; for any n you pick, the probability, on our evidence, that there are more than n such beings is 1. The medievals allegedly discussed the question: how many angels can dance on the point of a needle? They held that angels don't (ordinarily) take up any space, so that the answer is: as many as you please. This conclusion receives dramatic confirmation from the present account of probability: for any needle N and natural number n, the probability on any evidence that there are at least n angels dancing on the point of N is 1. (One can see why Carnap kept this consequence of his views under wraps.) From this point of view, Occam's Razor seems a singularly inappropriate piece of advice: one *can't* multiply entities beyond necessity.

But of course all this is absurd; and what it shows, as I see it, is that there isn't any probability function that accords both with intuition and with the probability calculus. The point is not, of course, that one cannot assign probabilities to the members of classes like H in such a way that this consequence does not ensue. Hintikka, for example, proposes a system of inductive logic in which universal and existential generalizations do not have *a priori* probability 0 and 1 respectively, even in an infinite domain.[27] Of course Hintikka is speaking of a system in which it is *sentences*, not propositions, to which *a priori* probability is assigned; and since this system is essentially a monadic predicate calculus without identity, only one of the propositions in H can be formulated in it. Hence Hintikka's system is at best of dubious relevance to our problem. Still, one can certainly assign *a priori* probability to the members of H in such a way that they do not all have zero *a priori* probability. One could, for example, assign zero *a priori* probability to all but a finite subclass C of H, evenly dividing up the probability among the members of C. You might think it quite out of the question, for example, that a possible world should have as many as 10^{20} horses (or angels); you might then assign 0 *a priori* probability to $H_{10^{20}}$ and all its successors, dividing up the probability equally among H_0 to $H_{10^{20}-1}$. You might also assign H_0 a probability of 1/2, H_1 1/4, H_2 1/8, and so on; or you might

assign the probabilities in accordance with some other appropriately convergent series.

The problem is, of course, that any of these methods involves assigning some members of H vastly greater probability on tautological evidence than others; if all get assigned some non-zero probability then for any number n you pick there will be a pair of members m and m^* of H such that the *a priori* probability of m is at least n times that of m^*. And of course any such method will be overwhelmingly biased against large numbers; for any member m you pick and any number n, there will be infinitely many larger members m^* of H such that the *a priori* probability of m is more than n times the *a priori* probability of m^*. And all of this is totally counter-intuitive. If the members of H *do* have probability on tautological evidence, none of them should be more probable than any other; on tautological evidence there are no numbers n and m such that m is more likely than n to be the number of horses.

If we think the members of H have *a priori* probabilities, we are faced with the following dilemma: either they have *differing a priori* probabilities, which seems quite wrong, or they have the *same a priori* probabilities, which also seems quite wrong. Fortunately we need not put up with either of these unpalatable alternatives; for we are not obliged to think that contingent propositions have any *a priori* probability *at all*.

The logical interpretation, therefore, confronts enormous difficulties. There is no reason to think contingent propositions *have a priori* probabilities; and there seems no plausible function to assign *a priori* probabilities. It is therefore hard to see how the atheologian could work out his atheological argument employing the logical interpretation of the probability calculus. Of course, even if this interpretation is in fact correct and (12*), G and E do have *a priori* probabilities, there isn't the slightest reason to think the *a priori* probability of (12*) is less than half that of E; hence there isn't the slightest reason to think that E disconfirms G.

VI. FREQUENCY AND ATHEOLOGY

Neither the personalist nor the logical interpretation have anything to offer the atheologian intent upon a probabilistic argument from evil. Let us therefore turn to the frequency interpretation of probability. I shall consider this interpretation in the form offered by Hans Reichenbach[28] and his out-

standing contemporary disciple, Wesley Salmon. The basic idea of the frequency interpretation begins with the notion that probabilities are ***ratios*** – proportions of events of one kind among events of another kind. So, for example, if we say that the probability of getting heads on a flip of this coin is 1/2, we are saying something about the proportion of heads among flips of this coin. Presumably it is not the *actual* tosses of this coin in which we are interested, however, for it could be that this coin will have been flipped only once during its entire existence, coming up heads that time; it should not follow that the probability of getting heads on a flip of this coin is 1. We mean to be speaking, somehow, of a series of *possible* flips. Less mythologically, though more counterfactually, we mean to be speaking of what *would* happen if this coin *were* flipped sufficiently often. (And perhaps what would happen is grounded or based, somehow, on a *propensity* the coin has to come up heads about as often as tails.[29])

How often is sufficiently often? If the coin were flipped an odd number of times, of course, the ratio of heads to flips couldn't be 1/2; more generally if the coin were flipped n times it couldn't be $1/n + 1$. But surely the probability of getting heads should not depend upon the number of tosses. That is, this probability should not depend upon n in

(52) If this coin were flipped n times, then the ratio of heads to flips would be m/n.

But of course if n is finite – i.e., is any natural number – there will be values m/n cannot assume. We must therefore speak of an infinite series of flips of the coin:

> According to a leading version of the frequency interpretation, probability is defined in terms of the limit of the relative frequency of an attribute in an infinite series of events. To say, for instance, that the probability of getting heads with this coin is one-half means that, in the potentially infinite sequence of tosses of the coin in question, the relative frequency with which heads occurs converges to the value of 1/2.[30]

To say that there is a potentially infinite sequence of tosses of this coin is to say, I gather, that the coin can be tossed infinitely often; and to say that the relative frequency of heads among tosses, in this series, converges to 1/2, is to say, presumably, that

(53) If this coin were flipped infinitely often, the limit of the relative frequency of heads among flips would be 1/2.

Here the notion of a limit has its standard mathematical interpretation, so that the consequent of (53) asserts that for any positive real number ϵ, there is an integer N such that for any integer N^* greater than N, the number of heads (H) in the first N^* flips is such that the difference between H/N^* and 1/2 is less than ϵ (that is, the proportion of heads among the first N^* tosses is within ϵ of 1/2). This, then, gives us the orthodox frequency interpretation of probability; $P(A/B) = n$ is the assertion that the limit of the relative frequency of type A events in a sequence of type B events is n.

Now of course there are some problems with this conception, even with respect to the very sorts of cases it is designed to accommodate. For one thing, this coin *can't* be flipped infinitely many times; it would wear out much too soon. But then perhaps we can speak of an infinite series of coins *like* this one is relevant respects. (Of course problems lurk here too; what are the relevant respects? Can they be specified independently of the probability of getting heads?) And then there is the fact that it is by no means easy to see what reason we might have for thinking that the series in question *has* a limit; maybe the series would diverge. But let's suppose for the moment that these problems can be satisfactorily resolved.

A. Frequency and the Probability of Propositions

There are other, more serious problems in trying to see how the frequency interpretation can be applied to our present concern: the atheological claim that $P(\mathrm{G/E}) < 1/2$. How shall we work out the frequency interpretation in such a way that it bears on this suggestion? G and E, after all, are *propositions*, not classes of events or objects. This is not a trivial problem. On the frequency interpretation, a statement of probability involves an infinite series of events and an attribute A possibly displayed by some of the members of the series. The probability statement then states that the relative frequency of events displaying A in S approaches some limiting value. And here there is no initial difficulty in specifying the relevant series of events. We naturally suppose that if this coin were tossed infinitely often, there would be a (temporally) first toss with outcome 0_1, a second toss with outcome 0_2, and so on. And the relevant series is just $0_1, 0_2, 0_3, \ldots$. (We might wonder whether either of the counterfactuals

(54) If this coin were tossed infinitely often, the 71st toss would come up heads

and

(55) If this coin were tossed infinitely often, the 71st toss would come up tails,

are *true*[31]; but let that pass.) The relevant series is just the (counterfactual) *temporal* series. Of course it is clear that if the coin were tossed infinitely often, the resulting outcomes could be arranged in some other order – e.g., 1 head, 2 tails, 4 heads, 8 tails, ... But this fact seems irrelevant; the right series is the one given by taking the outcomes in their temporal order.

But when we turn to the atheological claim that $P(G/E) < 1/2$, it is not easy to see, initially, how to understand or interpret it from the frequency point of view. What is the relevant infinite series of events? We shall have to *generalize* the frequency interpretation and take 'event' very broadly. If we are to apply the frequency theory, we must somehow associate classes C_1 and C_2 with G and E, in such a way that we can plausibly identify $P(G/E)$ with $P(C_1/C_2)$, i.e., with the relative frequency of members of C_1 among members of C_2. But how shall we do this? Is the idea that most universes in which there are 10^{13} turps of evil were not created by God? Of course, as Hume says in a similar connection, "To ascertain this reasoning it were requisite, that we had the experience of the origin of worlds"[32]; and as Charles Peirce says, worlds, i.e., universes, are not as plentiful as blackberries.

Possible worlds, however, are even more plentiful; so perhaps the atheological claim that $P(G/E) < 1/2$ is to be understood, somehow, as an assertion about the relation between the class of possible worlds in which G holds and the class of worlds which E holds. And presumably the claim will be that the limit of the relative frequency of G-worlds among E-worlds is less than 1/2. One wonders, of course, about the atheologian's source of information; where did he hear about how it goes in all those E-worlds? But there are other and more pressing problems. In particular, this suggestion faces two quasi-technical but extremely important problems.

(1) *Are there few enough possible worlds*? The first problem: probability, on the frequency theory, is defined in terms of the frequency of an attribute in a *sequence*; and a sequence has only countably many members – only as many members as there are natural numbers. But, for all we know, there may be *uncountably* many E-worlds – as many as there are *real* numbers. Perhaps,

for any real number r between 53 and 54, there is an E-world in which P. Q. Zwier's girth is r inches. If so, there are uncountably many E-worlds; and if there are uncountably many, they cannot be arranged in a sequence. But then there will be no such thing as the limit of the relative frequency of G-worlds among E-worlds. This suggestion, therefore, faces an unpleasant hurdle right from the start: how do we know that there are few enough E-worlds to form a sequence including them all?

(2) *Which sequence*? But suppose we agree that there are only countably many E-worlds; or suppose, *per impossible*, we settle on some method of picking out an appropriate countable subset S^* of the set of E-worlds, identifying $P(G/E)$ with the relative frequency of G-worlds among the members of S^*. Our problems are not at an end; in fact perhaps the most serious problem of all now looms. For of course there are *many* sequences associated with a countably infinite class C and a subset C^* of C (where a sequence is *associated* with C and C^* if it contains all the members of C^* and only the members of C). Typically only some of the associated sequences converge; and those that do typically converge to many *different* values. Suppose, for example, C and C^* are the classes of natural numbers and even natural numbers respectively. There will be some associated sequences in which the relative frequency of even numbers does not converge at all; for example

$$1, 2, 4, 3, 5, 7, 9, 6, 8, 10, 12, 14, 16, 18, 20 \ldots$$

where $n \frac{\text{odds}}{\text{evens}}$ are followed by $2n \frac{\text{evens}}{\text{odds}}$. Of course many of the associated sequences do converge; for example

$$2, 3, 5, 4, 7, 9, 6, 11, 13 \ldots$$

where each even number is followed by two odds. Here the limit of the relative frequency of evens among integers is 1/3. If we exchange evens for odds throughout we get another sequence in which the relative frequency of evens converges to 2/3. Indeed, for any real number r between 0 and 1 (including those end points) there is an associated sequence such that the limit of the relative frequency of even natural numbers among natural numbers in that sequence is n.

But then the same will go for G-worlds and E-worlds. Of course if there are only finitely many G-worlds among the E-worlds, then for any associated sequence the relative frequency of G-worlds among E-worlds will converge

to 0; and if there are only finitely many G-worlds among the E-worlds, the relative frequency of G-worlds among E-worlds, for any associated sequence, will converge to 1. But if there are any G-worlds at all among the E-worlds, then there will undoubtedly be infinitely many; if there is a G-and-E world in which Zwier's girth is 54½ inches, then for any rational number r between 54 and 55 there will be a G-and-E-world-where Zwier's girth is r inches. So there will be infinitely many G-worlds among the E-worlds. And if so, then for any real number you pick between 0 and 1 there will be an associated sequence such that the limit of G-worlds among E-worlds for that sequence is r. How do we decide, then, which sequence should be picked to determine P(G/E)? In the coin tossing case, as we saw, the natural course is to take the *temporally* ordered sequence of outcomes that would result if the coin were tossed infinitely often; but of course possible worlds aren't temporally ordered. So which sequence do we pick? Does the theist get his choice? If so, he'll no doubt pick one in which the limit is 1, in which case the atheologian will be tugging the laboring oar. On the other hand, if the atheologian gets *his* choice, he'll no doubt pick a sequence where the limit is zero. So which sequence shall we pick?

Here there is a real problem in using the frequency interpretation to understand P(G/E): granted that there are only countably many E-worlds, which of the many associated sequences is the right one to pick to determine P(G/E)? This problem is easy to overlook; Wesley Salmon, for example, writes as if there is typically such a thing as *the* limit of the relative frequency of A's among B's, forgetting that such a limit is always relative to a sequence. Thus in defining *statistical relevance* he writes,

> Suppose we ask for the probability that a given individual x has a characteristic B. We know that x belongs to a reference class A in which the limit of the relative frequency of B is p. If we can find a property C in terms of which the reference class A can be split, ... (*Foundations*, p. 91)

But the notion of the limit of a relative frequency makes sense only with respect to a (countably) infinite sequence; so A must be a countably infinite class. If there are only finitely many B's in A (or only finitely many $\overline{B}$'s), then the appropriate limit, as we have seen, will be zero (or one) for any associated sequence; but if there are infinitely many B's among the A's, then there will be many different associated sequences, so that for any real number r between zero and one there will be a sequence in which the appropriate limit is r.

Our problem, therefore, remains: how shall we appropriately associate a class and attribute with G and E in such a way that we can speak of the probability, in the frequency sense, of G with respect to E?

B. Salmon's Suggestion, I

Salmon makes an impressive effort to answer this question. The key, he thinks, is Bayes' Theorem:

> ...things get much more complex when we try to apply Bayes' theorem to genuine scientific hypotheses. Serious problems of interpretation arise. I shall claim, nevertheless, that Bayes' theorem provides the appropriate logical schema to characterize inference designed to establish scientific hypotheses. The hypothetico-deductive method is, I think, an over-simplification of Bayes' theorem. It is fallacious as it stands, but it can be rectified by supplementing it with the remaining elements required for application of Bayes' theorem. (*Foundations*, p. 117)

Salmon invites us to consider a couple of forms of Bayes' Theorem[33]:

$$(56) \qquad P(B/(A \,\&\, C)) = \frac{P(B/A) \times P(C/(A \,\&\, B))}{P(C/A)}$$

and

$$(57) \qquad P(B/(A \,\&\, C)) = \frac{P(B/A) \times P(C/(A \,\&\, B))}{P(B/A) \times P(C/(A \,\&\, B)) + P(\bar{B}/A) \times P(C/(A \,\&\, \bar{B}))}$$

He then continues:

> Let us, therefore, compare the hypothetico-deductive method with Bayes' theorem. From an hypothesis *H* and statements of initial conditions *I*, an observational prediction *O* is deducible. For purposes of this discussion we assume I to be true and unproblematic. Under this assumption *H* implies *O*. We can provide a loose and preliminary interpretation of Bayes' theorem, even though many difficult problems of interpretation remain to be discussed. Let 'A': refer to hypotheses like *H*; let '*B*' refer to the property of truth; and let 'C' refer to the observed result with respect to the prediction *O*. If positive confirmation occurs 'C' means that *O* obtains; in the negative case 'C' designates the falsity of *O*. This interpretation makes the expression on the left-hand side of Bayes' theorem refer to precisely the sort of probability that interests us; '*P*(*B*/(*A* & *C*))' designates the probability that a hypothesis of the sort in question, for which we have found the given observational result, is true. This is the probability we are looking for when we deal with the confirmation of scientific hypotheses. (*Foundations*, p. 117)

Let's try to understand this suggestion. Since it is a *frequency* interpretation, *A*, *B* and *C* must be classes, or attributes determining a class. *B* is the attribute of truth; no problem there. *A* is the class of hypotheses 'like' *H*; the class of hypotheses that in relevant but so far unspecified respects resemble *H*. It is considerably more difficult to see what *C* might be. "...let

'C' refer to the observed result with respect to the prediction *0*. If positive confirmation occurs, 'C' means that *0* obtains; in the negative case 'C' designates the falsity of *0*." But *C* must be a *class* or an *attribute*: what class or attribute is it? It isn't easy to tell; apparently it is *one* class or attribute if *0* is true and a *different* class or attribute if *0* is false. One might expect *C* to follow the lead of *A*, which is the class of H-like hypotheses; then *C* would be the class of propositions *like 0*. But apparently this is not what Salmon has in mind. Perhaps there is an appropriate interpretation lurking in the neighborhood, but it is hard to see what it might be.

When we turn to trying to understand Salmon's interpretation of the various terms in Bayes' theorem, there are further perplexities. Consider the right hand side of (56); it contains two prior probabilities ($P(B/A)$ and $P(C/A)$), and also the likelihood $P(C/(A \,\&\, B))$. The latter need give us no trouble in the present context; we're interested in the sort of case where *0* is entailed by *H*, so that whatever the interpretation of that term, its value will be 1. The prior probability in the numerator, says Salmon can be interpreted as follows:

> In the effort to show how the frequency concept of probability can be made relevant to the probability of hypotheses through the use of Bayes' theorem, we must find the appropriate prior probability $P(B/A)$. This is the probability that hypotheses of a certain type are true. The attribute of truth is given directly by the fact that we are looking for true hypotheses. In attempting to choose an appropriate reference class, we are trying to find out what type of hypothesis is likely to be true.... A hypothesis that belongs to the class of plausible conjectures is one that has a high prior probability. One that belongs to the class of preposterous conjectures is one that has a vanishingly small prior probability.... To evaluate a given hypothesis *H*, we try to find a (practically or epistemically) homogeneous reference class *A* to which *H* belongs. *A* must be a class of hypotheses within which we can say something about the relative frequency of truth. The probability $P(B/A)$ is the probability of truth for hypotheses of this class, and this probability is assigned as a weight to the hypothesis *H*. This weight, which might be distinguished as a prior weight, expresses the plausibility of *H*. (*Foundations*, p. 124)

This gives us a fairly clear account of the prior probability in the numerator of Bayes' Theorem: it is something like the relative frequency of truth among hypotheses like *H*. (Of course, here we must remember an earlier *caveat*: if the class of hypotheses like *H* is infinite, there will not, in general, be any such thing as *the* relative frequency of truth among members of this class.) But when we come to the prior probability in the denominator of (56), I cannot make out what Salmon has in mind:

> The denominator of Bayes' theorem can be written in either of two ways, because of the theorem on total probability. The simpler form involves $P(C/A)$, the probability of obtaining the observational result regardless of the truth of our hypothesis is *H* (*Foundations*, p. 118).

$P(C/A)$ *should* give us the prior probability of C, i.e., the probability of obtaining the observational result 0 regardless of the truth of H; but how could we interpret $P(C/A)$ that way? A is the class of hypotheses like H. C as we have seen, has problems of its own; but no matter how, exactly, we understand C, $P(C/A)$ would have to represent the frequency of some attribute related to 0 among hypotheses like A. And how could we possibly see that as the probability of 0 regardless of the truth of H? I find it hard to understand Salmon's proposal for explaining the idea of the probability of a scientific hypothesis from the point of view of the frequency theory of probability.

C. *Salmon's Suggestion, II*

But suppose we look into what Salmon's suggestion suggests. Consider the following form of Bayes' Theorem:

$$(58) \qquad P(A/(B \,\&\, T)) = \frac{P(A/T) \times P(B/(A \,\&\, T))}{P(B/T)}$$

(here T is any tautology). If we abbreviate '$P(A/T)$' and '$P(B/T)$' to '$P(A)$' and '$P(B)$' and use A_5, we can rewrite (56) as

$$(59) \qquad P(A/B) = \frac{P(A) \times P(B/A)}{P(B)}.$$

Now suppose we limit consideration to the sort of case where A entails B, so that $P(B/A) = 1$. Then following Salmon's lead, we may see the prior probabilities $P(A)$ and $P(B)$ as representing the frequency of truth among propositions like A and B. Of course there is the (by now) familiar problem that a relative frequency involves not only an attribute and a class but also a *sequence*, so that in general there is no such thing as *the* relative frequency of the attribute in the class. But let us suppose that for each attribute and class (or at least for truth and each class of propositions *resembling* a given proposition), an appropriate associated sequence is somehow given or selected. Then $P(A)$ will be the limit of the relative frequency of truth in the appropriate sequence associated with truth and the class of A-like propositions; call it 'the limit of truth among A-like propositions'. $P(B)$ will be the limit of truth among B-like propositions; and $P(A/B)$ will be the limit of truth

among propositions like $A \& B$ divided by the limit of truth among B-like propositions – that is, $P(A \& B)/P(B)$. This gives us a simple frequency interpretation of Bayes' Theorem very much in the spirit of Salmon. (And of course the interpretation is easily extended to the probability calculus generally.)

For the sake of concreteness, let us briefly return to the problem of evil. Recall

(12*) God is omnipotent, omniscient and wholly good, and God could not have actualized a world as good as the actual world but containing less evil than the 10^{13} turps of evil the actual world contains;

and recall that the atheologian is committed to holding that $P((12^*)/\mathrm{E}) < 1/2$ (since he holds both that $P(\mathrm{G}/\mathrm{E}) < 1/2$ and that (12*) entails G). By (59), then,

$$(60) \qquad P((12^*)/\mathrm{E}) = \frac{P(12^*) \times P(\mathrm{E}/(12^*))}{P(E)},$$

and since (12*) entails E, (60) is equivalent to

$$(61) \qquad P((12^*)/\mathrm{E}) = \frac{P(12^*)}{P(\mathrm{E})}.$$

From the present perspective, then, the atheological claim is that the relative frequency of truth among propositions like (12*) is less than 1/2 the relative frequency of truth among propositions like E.

D. Some Problems

This claim and this interpretation are obviously not without problems. Can we really suppose that there is an appropriate sequence associated with truth and the class of propositions like (12*), or G? For any real number between zero and 1 there will be a sequence of propositions like (12*) in which the limit of the relative frequency of truth is r; what, then, determines that one of these is the appropriate one and gives us what can plausibly be taken to be the probability of truth among propositions like (12*)? But even more pressing is the following problem: what is the class of propositions *like* G or E or (12*)? How are we to decide which propositions relevantly resemble

them? G, for example, is a member of *many* impressive classes: the class of theological propositions, of propositions believed by many people, of existential propositions, of propositions entailing the existence of a *person* of a certain sort, of controversial propositions, important propositions, and the like. Which, of these, if any, is the class of propositions relevantly like G? In which reference class shall we put G – or, for that matter, E or (12*)?

Salmon recognizes the problem; in essence, he says, it is the problem of the single case – a problem of considerable concern for frequentists. Suppose we are interested in the probability that Feike can swim. We may know that Feike is a Frisian and that 9 out of 10 Frisians can swim, that he is a farmer and 6 out of 10 farmers can swim, that he is an octogenarian and 3 out of 10 octogenarians can swim, that he lives on the North Sea and 19 out of 20 North Sea coast dwellers can swim, and the like. How shall we decide what the likelihood is that Feike can swim? Which of these various classes is the one to choose?

The answer, says Salmon, is to choose the *broadest homogenous reference class.*

> The specification of the attribute class is of no particular difficulty for the frequentist attempting to apply probability to the single case; the whole difficulty rests with the selection of the reference class. The rule is to select the broadest homogenous reference class available.... To evaluate [the prior probability of] a given hypothesis H, we try to find a (practically or epistemically) homogeneous reference class A to which H belongs. A must be a class of hypotheses within which we can say something about the relative frequency of truth. (*Foundations*, p. 124)

The notion of homogeneity, for a class, is explained as follows:

> I would say ... that the single case should be referred to the broadest homogenous reference class of which it is a member.... A probability is something that has to be established inductively, and in order to ascertain the probability we must have enough instances to be able to make an inductive generalization. Thus we do not want to try to refer single cases to classes that are too narrow, for if we do we will not have enough evidence upon which to base our inference. At the same time, we want our reference class to contain other relevant cases, not irrelevant ones. Statistical relevance is the key concept here. Suppose we ask for the probability that a given individual x has a characteristic B. We know that x belongs to a reference class A in which the limit of the relative frequency of B is p. If we can find a property C in terms of which the reference class A can be split into two parts $A \,\&\, C$ and $A \,\&\, \overline{C}$, such that
>
> $$P(B/(A \,\&\, C)) \neq P(B/A),$$
>
> then C is statistically relevant to the occurrence of B within A. Of course, C must be the sort of property whose occurrence in an individual can be detected without knowing whether that particular entity also has the property B. If there is no such property C by means of which to effect a relevant subdivision of A with respect to the occurrence of B, A is homogeneous with respect to B....

> It would be most unrealistic to suppose that we can fulfill the requirement of selecting the broadest homogenous reference class in all cases in which we may have to make practical decisions about single events. We may suspect that a given reference class is inhomogenous, but not know of any way to make a relevant partition of it. Under these circumstances let us say that the class is *epistemically homogenous*; reference classes of this type are the best we have in many cases... .

By way of brief recapitulation: to understand the atheological claim from the frequentist point of view, we must employ Bayes' Theorem. To find the appropriate prior probabilities we must find the classes of propositions like G, E and (12*). These classes are the broadest homogenous reference classes containing the propositions in question; and the relevant prior probabilities are the limits of the relative frequency of truth in those classes. We are temporarily conceding that for each of these reference classes and the attribute truth, an appropriate associated sequence is somehow given or selected.

Return, by way of illustration, to Feike and the probability that he can swim; and suppose we try to follow Salmon's directions for picking the relevant reference class. The right reference class is the broadest class that contains Feike and is homogeneous with respect to capability of swimming; but what class *is* that? The class of North Sea dwellers, or the class of Frisians? These seem appropriate choices; unfortunately, however, there are at best finitely many Frisians. So there aren't enough for a sequence. Presumably, then, we must follow the same course here as in the coin tossing case; we must consider what things would be like if there were *infinitely* many Frisians. By way of picking an appropriate sequence, perhaps we could take them alphabetically: Aardsma, Bouwsma, Cammenga, De Vries, ..., Frisians of the same name being ordered according to weight, let's say. This class would then be an appropriate reference class for the present inquiry, provided it were homogenous; provided, that is, that no attribute picked out a (potentially infinite) subclass of Frisians in which the relative frequency of truth (with respect to a similarly constructed sequence) differed from that in the parent class.

Of course there are some thorny problems here. If there were infinitely many Frisians, there might be room for little else – very little water, say. So how would any of these Frisians have learned to swim? Or is it that we aren't to suppose that infinitely many Frisians exist at once, but only that for any number n you pick, there is a time t such that by t God will have had the goodness to create n Frisians? In either case, however, it seems totally paradoxical to suppose that the likelihood that Feike can swim depends

on what things would be like if there were infinitely many Frisians. Suppose the fact is there are only finitely many: then if there were infinitely many, the world would be *toto caelo* different from what it is – so different, in fact, that consideration of what it would be like, under those conditions, seems quite irrelevant to determining the probability that Feike can swim.

So there are some real problems with the frequentist's basic insight. Salmon's definition of homogeneity is also not without problems. It appears to go something like

(62) A is homogeneous with respect to attribute B if there is no attribute C such that (1) we can detect the occurrence of C in an individual without knowing whether it has B, and (2) the relative frequency of B in the class determined by the conjunctive attribute A & C differs from that of B in A.

The 'can', I take it, refers to our *de facto* abilities and not merely to what is logically possible. Of course people may differ as to whether they can detect the occurrence of C in an individual without knowing whether it has B; perhaps we must relativize homogeneity to *persons*. And is the idea that C is to be such that for *any* object x, we can determine whether or not x has C without knowing whether it has B? This will make the set of classes homogeneous with respect to B too broad. A class will then be inhomogenous with respect to B only if there is some attribute C such that for any object x, we can determine whether x has C without determining whether it has B; but there are very few interesting attributes C such that for *just any* object x we can determine whether x has C at all – let alone determine whether x has C without determining whether x has some other attribute B.

Is the idea that the attribute C is to be such that for *some* object x, we can determine whether x has C without determining whether it has B? But then (except where there are only finitely many B's or $\overline{B}$'s in A) there won't be any homogeneous reference classes at all; there will always be a class C in terms of which to effect the partition. Let C^* be any finite set of objects – the set of Frisians, for example. Suppose the members of C^* are C_1, C_2 ... C_n; and consider the attributes

A_1 is identical with C_1, or is identical with C_2, or ...or is identical with C_n, or has B

and

A_2 is identical with C_1 or is identical with C_2 or ...or is identical with C_n or has $\overline{B}$.

For some objects x (in particular, for Frisians) it will be possible to tell whether x has A_1 or A_2 without knowing whether x has B. If the relative frequency of B in A is other than 1, A_2 will be an appropriate C to show that A is not homogeneous with respect to B; for the relative frequency of B in the class determined by A_1 & A is 1 (since all but finitely many members of that class have B). On the other hand, if the relative frequency of B in A is 1, then A_2 will be an appropriate C; the relative frequency of B in the class determined by A_2 & A will be zero, since all but finitely many members of that class *lack B*.

Perhaps the idea is that C is to be such that any object x of which we can determine whether it has C, is such that we can determine whether it has C without determining whether it has B. There are problems here too, but we may ignore them for the moment.

Suppose we waive all these problems and pretend we know what the appropriate reference class for G (and E and (12^*)) is. Now how are we to estimate the relative frequency of truth in that class? We imagine that for each such class there is an appropriate converging associated sequence for truth and the class in question; how shall we determine the value to which it converges? Salmon makes the following suggestion:

> In the frequency interpretation, as I am treating it, synthetic probability statements are asserted on the basis of inductive evidence. This approach allows the use of induction by enumeration, or some other asymptotic method, to infer the limit of the relative frequency – i.e., the probability – from the observed frequency in a finite sample. (*Foundations*, p. 95)

What he means, I think, is the following. To estimate the relative frequency of truth in the class C of propositions like G, we check to see what the proportion of true propositions is among the members of our *sample* of this class; the set of propositions whose members are members of C and whose truth value we already know. So we first settle on the appropriate reference class R; we then consider that subset C of R such that we know of each member of C whether or not it is true; this is our sample of R. And then we identify the relative frequency of truth in R with the proportion of truths in our sample C. From a practical point of view, this means that we can sidestep the difficulty about the many sequences associated with R; we can take the appropriate associated sequence to be any sequence that converges

to the proportion of true propositions among those members of this class whose truth value we know. And of course the same will go for (12^*) and E.

By way of recapitulation, then: the atheologian holds that $P(G/E) < 1/2$ and hence that $P(12)^*/E)) < 1/2$. By Bayes' Theorem,

$$(63) \qquad P(G/E) = \frac{P(G) \times P(E/G)}{P(E)}$$

and

$$(64) \qquad P((12^*)/E) = \frac{P(12^*) \times P(E/(12^*))}{P(E)}$$

On our reconstructed Salmonian application of the frequency interpretation, we estimate each of $P(G)$, $P(12^*)$ and $P(E)$ by picking the broadest homogeneous reference classes containing it; we then determine the value in question by determining the proportion of true propositions among those in the class in question whose truth value we know.

VII. RELATIVITY TO NOETIC STRUCTURES

The Salmonian frequency interpretation throws into sharp relief one important feature of the situation we are exploring: what I properly or justifiably take to be the probability of G or (12^*) on E will crucially depend upon what *else* I believe or know or think I know. Let us say that a person *S*'s *noetic structure* is the set of propositions he believes, together with various logical and epistemic relations among these propositions. Thus a noetic structure contains an index of degree of belief, indicating how *firmly S* believes each proposition in S^*; it indicates which propositions *S* believes on the basis of evidence conferred by other propositions and which he accepts without accepting them on the basis of evidence, and the like. Then we may say that what I properly or justifiably take to be the probability of G or (12^*) with respect to E depends upon my noetic structure. Thus different people can perfectly properly come to different views on this matter.

This is abundantly evident with respect to our Salmonian version of the frequency interpretation. To estimate $P(G)$ and $P(E)$, we are directed to pick the broadest homogeneous reference classes containing these propositions. But what, for G, for example, will this class be? The theist, who thinks that G is in fact true, should pick the class of *true propositions* as the appropriate

reference class. That class is certainly homogeneous with respect to truth; and he presumably won't know of one much broader that is homogeneous with respect to truth. And, of course, the limit of the relative frequency of truth in any sequence of true propositions is 1. So he should take it that the prior probability of G is 1, in which case the same thing will go for its posterior probability on E.

Let me repeat and emphasize this point. The theist is attempting to determine whether $P(G/E) < 1/2$, as the atheologian claims. According to the present scheme, we apply Bayes' Theorem

$$P(G/E) = \frac{P(G) \times P(E/G)}{P(E)}.$$

$P(G)$ represents the relative frequency of truth in the class of propositions relevantly like G: according to Salmon, the broadest homogeneous reference class. Which class *is* that class? What class should the theist pick? Of course it isn't as if he has no notion of what the truth value of G is: being a theist, he takes it, naturally enough, to be *true*. But then he takes G to be a member of the class of true propositions. That class is trivially homogenous with respect to truth in Salmon's sense; there is no attribute *C* such that the relative frequency of truth in the class determined by the conjunctive attribute *C and truth* differs from that of truth in the class of true propositions. Furthermore, it is hard to see how one could find a *broader* class homogeneous with respect to truth. He should therefore take the class of true propositions to be the broadest reference class for G homogeneous with respect to truth. But of course the proportion of true propositions in our sample of true propositions is 1; following Salmon's suggestion, he should therefore assign G a prior probability of 1. But then, naturally enough, its probability on E will also be 1.

Of course the atheist will be equally within *his* rights in concluding, by a similar argument, that $P(G) = 0$, so that $P(G/E) = 0$. That theist and atheist may justifiably assign $P(G/E)$ such different values, using this method, simply reflects their intial disagreement with respect to G. But then this method doesn't resolve or tend to resolve the issue between them; nor does it tend to show that one or the other is suffering from some noetic deficiency. The important point here is that a method like the one Salmon proposes will not be polemically useful.

So on the Salmonian reconstruction of the frequency interpretation, what

I should take as the probability of G on E crucially depends upon my noetic structure. But even apart from the specific details of this proposal, it is clear that a person's estimate of the probability of G on E should depend on the rest of his beliefs. Let me illustrate. The Free Will Defense, in at least some versions, makes reference to demons, or other non-human free spirits. Thus consider, for example,

(65) All natural evil is due to the free activity of non-human persons; there is a balance of good over evil with respect to the actions of these non-human persons; and there is no world God could have created which contains a more favorable balance of good over evil with respect to the free activity of the non-human persons it contains.[34]

(65) plays r in

$$[\Diamond(p \,\&\, r) \,\&\, ((p \,\&\, r) \rightarrow q)] \rightarrow \Diamond(p \,\&\, q),$$

where G plays p and *there is natural evil* plays q. (65) has sometimes been criticized on the grounds that it entails the existence of demons and/or other non-human persons and is thus monumentally improbable. (Of course (65) needn't be *true* or *probable* for the purposes of the Free Will Defense; it need only be consistent with G and in conjunction with it entail that there is natural evil; but let that pass.) But why suppose (65) *is* improbable? Surely it isn't particularly improbable with respect to a *theistic* noetic structure. The theist already believes that there are non-human persons (God, for example); and certainly has no reason to think God couldn't or wouldn't create more non-human persons who play a substantial historical role. So he has no reason at all to hold that (65) has low prior or antecedent probability; indeed, he may believe that (65) is just the sort of hypothesis that might well be correct. The atheist on the other hand, might think (65) quite improbable, antecedently.

But why suppose theist and atheist ought to agree here? If they disagree, neither need be in error; for they are estimating the probability of (65) with respect to very different noetic structures. Theist and non-theist differ, of course, about the existence of God; but they are likely to differ at many other points as well. They may differ, for example, with respect to the proportions of good and evil in the universe at large; the Christian theist will no doubt concur with St. Paul: "For I reckon that the sufferings we now

endure bear no comparison with the splendor, as yet unrevealed, which is in store for us". (*Romans* 8: 18) They may also disagree as to the *extent* or *amount* of good the universe contains. From a Christian point of view, there is immortality and the expectation of a better world; and, towering above all, the unthinkable splendor of God's gift to mankind in the suffering and death and resurrection of His Son. It is therefore no wonder theist and non-theist differ about prior probabilities.

Another example: Consider

(12*) God is the omnipotent, omniscient and wholly good creator of the world; and every world that God could have actualized that contains less than 10^{13} turps of evil contains a less favorable overall balance of good and evil than does the actual world.

The atheologian who holds that $P(G/E) < 1/2$ must hold that the prior probability of (12*) is less than one-half that of E. But here again the theist and atheist may sensibly disagree; what one correctly takes the plausibility or prior probability of (12*) to be will depend on what else one believes. The theist thinks its left conjunct is true and that there are 10^{13} turps of evil; perhaps he thus has a reason for thinking $P(12^*)$ fairly high – a reason the non-theist won't have. The atheologian, on the other hand, may estimate this probability as fairly low. This is as it should be; as I argued earlier, there are no interesting prior probabilities *simpliciter* – i.e., no interesting *a priori* probabilities.

Accordingly, this consequence of the Salmonian application of the frequency theory seems quite right. On the logical view, to be sure, a contingent proposition has an *a priori* probability that is entirely independent of anyone's knowledge or belief; as we have seen however, there isn't even the slightest reason to think such propositions have such probabilities, and good reason to think they don't. And both personalist and reconstructed frequential views properly eschew such *a priori* probabilities, replacing them by probabilities prior to a given investigation, or given inquiry, or given application of Bayes' Theorem; and these prior probabilities are relative to the noetic structure of the inquirer.

VIII. EVIL AND FOUNDATIONALISM

What we have seen so far is that none of the main interpretations of prob-

ability provide the atheologian with resources for a decent objection to theism based on the premiss that $P(G/E)$ is low. On the logical view, this claim involves extremely dubious suggestions about the probability on tautologies of G, E, and (12^*) – suggestions that were doubly dubious, since in the first place there is good reason to deny that such propositions *have* such probability on such evidence, and, in the second, even if they did there isn't the slightest reason to think these *a priori* probabilities have the values the atheologian says they have. The personalist view, on the other hand, makes no such dubious claims: but on that view '$P(G/E) < 1/2$' carries an implicit subscript and simply measures $P(G \& E)/P(E)$ for some person or other – perhaps the atheologian. But then it constitutes a piece of atheological autobiography rather than an objection to theism. The advocate of the frequency view encounters substantial difficulty in construing probability as a relationship among propositions in frequentist terms. If, however, we try to work out such an interpretation, keeping to the spirit of frequentism, we find that prior probabilities in an application of Bayes' Theorem are to be assigned in the light of what one already knows or believes. Here, then, just as with personalism, prior probabilities are relative to noetic structures. Of course there is no reason to think theist and atheist need agree about these prior probabilities; there is therefore no reason to think they either will or should agree about the value of $P(G/E)$.

But this has important consequences for the atheologian who offers the inductive argument from evil as an objection to theism, as a reason for supposing that the theist's views are somehow rationally objectionable, or irrational, or noetically below par. For what it shows is that this atheological program is totally misconceived. If prior probabilities are thus relative to noetic structures, it is no wonder that theist and atheologian will assign different values to $P(G)$ and $P(E)$; and hence it is no more than a bit of intellectual imperialism for the atheologian to insist that the theist accept the atheological estimate of $P(G/E)$. One who offers this sort of inductive argument as an objection to theism is like a theist, who offers as an objection to atheism, the fact that we all agree that there are 10^{13} turps of evil, together with the claim that the probability of G with respect to that evidence is high. What atheist should pay any attention to that? Such an argument of course, is thoroughly silly; but from the present perspective the atheolotical argumen from evil is really no more sensible.

Now perhaps the following objection might be tempting as against my line

of argument: "You simply exploit the difficulties inherent in the current analyses of probability to urge a sort of skepticism about probability claims. If you were right, we could never properly argue that a proposition ought not to be accepted on the grounds that it is improbable with respect to what we know – our total evidence, say. But often we *do* know that a proposition is improbable with respect to our total evidence and on that account ought not to be accepted; and the difficulties you raise are neither here nor there."

This objection misses the mark. I have no quarrel with the personalist account, or with attempts like Salmon's to apply the frequency theory to the probability relation as it holds between propositions; perhaps some such attempt can be made to work. Furthermore, I wouldn't dream of denying that there are cases where a proposition A is quite clearly improbable with respect to another proposition B or quite clearly improbable with respect to our total evidence. But the present discussion is not about one of these cases. It isn't initially in the least obvious that $P(G/E)$ is low, or that the prior probability of (12^*) is less than a half that of E, or that G or (12^*) are improbable on any body of knowledge the atheologian could plausibly propose as the relevant body of total evidence. None of these things is self-evident, or obvious (although as Kriesel says, it isn't obvious what's obvious) or accepted by all rational persons who think about it, or anything of that kind. So this objection misses the mark.

Of course there are contexts where arguments formally similar to the atheological probabilistic argument from evil are entirely in order. These are contexts where there is some basic body B of knowledge or belief accepted by all parties to the discussion, and where it is explicitly or implicitly agreed that the acceptability of a given proposition A depends upon its relationship to B – perhaps its probability with respect to B, or perhaps its capacity to explain some significant segment of B. This situation sometimes arises in a court of law, and in relatively uncontroversial areas of a well-established science; it also arises in the context of every daylife. As we have seen, however, there is no such agreed-upon basic body of knowledge B relevant to the present discussion. So the atheological probabilistic argument from evil is totally misconceived.

And this leads us beyond the argument from evil to a set of issues that are both broader and murkier. By way of conclusion I want to make a couple of sketchy and programmatic remarks about these issues. First, it is often said that religious beliefs – the beliefs of Christian, Jew or Moslem, for example –

should be *tested by the evidence*; such a set of beliefs is properly or rationally accepted only if it properly accords with the evidence.[35] But *which* evidence? What *is* the evidence? Well, maybe the following. There is a set *B* of propositions we all or nearly all accept: self-evident propositions and the propositions of common sense, perhaps, together with a host of perceptual beliefs. Perhaps the deliverances of the science, or at least some of the deliverances of some of the sciences will also be said to belong in *B*, along with, perhaps, propositions known to be entailed by any of the above, or in some other way establishable by reference to them. These are the things we know; this is the realm of knowledge. Whatever else a person believes – his political views, perhaps, his views on ethical and esthetic matters, his religious allegiances – is a matter of opinion, *faith* – as opposed to knowledge. And a religious faith is *rational*, or rationally acceptable, only if it is probable with respect to *B*, or at any rate not improbable with respect to it. Religious faith, therefore, must be tested by the evidence; it must be evaluated by reference to *B*. And perhaps from this point of view the atheological argument from evil is an appropriate approach to the question of the rationality of theistic belief. For from this point of view theistic belief will be reasonable only if it is probable or not improbable with respect to *B*; and of course *B* will contain the proposition that there is a great deal of evil. Furthermore, the prior probabilities of Bayes' Theorem will be estimated with respect to *B*, so that we might expect that honest and careful theists and atheologians will at least roughly concur on them. I don't really see how the relevant prior probabilities can sensibly be estimated; it looks to me as if it would be anybody's guess what they were; but at least from this point of view the atheological attempt makes a certain initial sense.

This is the conception of faith and reason perceptively attacked by such Reformed or Calvinist thinkers as Abraham Kuyper[36]; he insists on

> the utter untenability of the current representation that *science* establishes truth, which is equally binding upon all, exclusively on the grounds of observation and demonstration, while *faith* is in order only in the realm of suppositions and uncertainties. (p. 142)

On the conception Kuyper attacks, religious faith is or is relevantly like a *hypothesis,* designed, perhaps, to *explain* some part of *B*, or at any rate acceptable only if probable with respect to *B*. On this view there is a common foundation of belief shared by all or nearly all reasonable persons; and the question of the rational acceptability of religious belief is the question whether it is probable with respect to this foundation.[37]

Now from this point of view, most of the beliefs characteristically accepted by, say, Christians are peculiarly ill-founded. How could the existence of a triune God, or the incarnation of the Second Person of the Trinity, or the death and resurrection of the Son of God be sensibly thought of as *hypotheses* designed to explain what we find in B? What, in B do these things explain? Could one sensibly claim they are worthy of belief because of their high probability with respect to B, or because they explain some significant portion of B? Obviously not.

But now for a question of great importance: why suppose theistic belief must be probable with respect to B, to be rationally acceptable? What's so special about B? B, we are told, contains the propositions I *know*, the propositions with respect to which I am to judge the acceptability of other propositions. How do propositions get into B anyway? B contains propositions implying that there are other persons, that the world is more than 10 minutes old, and that there are material objects that persist when no one is looking. What makes these good candidates for being in B? Clearly there is no way they can be proved or established with respect to the sorts of propositions either classical or modern foundationalists are prepared to accept in the foundations of a rational noetic structure: that is the central lesson of the development of modern philosophy from Descartes to Hume. So how are they epistemologically superior to, say, the central tenets of Christianity?

Suppose we say that the *foundation* of S's noetic structure is the set of propositions S accepts but does not accept on the basis of other propositions; the foundation of my noetic structure is the set of propositions I start with, we might say; and I properly judge the acceptability of other propositions by their relation to those in this set. And now the central question is this: why shouldn't the existence of God be in the foundations of my noetic structure? Why can't I properly take G to be a member of this set, so that it is one of the propositions with respect to which the plausibility or acceptability of *other* propositions – propositions about angels and demons and other non-human persons, for example, – is to be judged? What propositions can properly go into the foundation of a noetic structure? How do we decide what sort of foundation a rational noetic structure can have? What are the rules here? Who promulgates them? *Are* there rules here? These are some of the questions to which this inquiry leads. But these are questions for another time.[38]

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NOTES

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[1] F. H. Bradley, J. M. E. McTaggert and J. S. Mill have made this claim as, more recently, have J. L. Mackie, H. J. McCoskey, H. D. Aiken and W. Kaufmann; John Wisdom is an exception, he held that the existence of God is logically *consistent* with that of evil, but improbable with respect to the amount of evil the world in fact displays (*Mind*, 1930).

[2] 'God and Evil', *Philosophical Quarterly*, 1960, 97.

[3] *The Faith of a Heretic*, Doubleday, Inc., New York, 1961, pp. 151–152.

[4] See, for example, J. Cornman and K. Lehrer, *Philosophical Problems and Arguments* (MacMillan & Co., New York, 1974); E. Madden and P. Hare, *Evil and the Concept of God* (Charles C. Thomas, Publ., Springfield Illinois, 1968); and W. Rowe, *Philosophy of Religion* (Dickenson Publ. Co., Encino, California, June, 1978).

[5] *The Nature of Necessity* (Clarendon Press, Oxford, 1974), pp. 164–193. (Hereafter 'NN').

[6] *Op. Cit.*, p. 194.

[7] *Philosophical Problems and Arguments*, pp. 340–341.

[8] *NN*, pp. 180–184.

[9] APA Eastern Division Symposium, Dec. 1977. The same passage may be found in Rowe's *Philsophy of Religion*, p. 89.

[10] Where a state of affairs S includes a state of affairs S^* if and only if it is not possible that S obtain and S^* fail to obtain.

[11] In the sense of *weakly actualize* as explained in *NN*, pp. 172–173.

[12] But not all; see, e.g., L. J. Cohen, *The Provable and the Probable* (Oxford University Press, Oxford, 1978).

[13] Here used autonymously.

[14] These axioms are adapted from Carnap.

[15] For fuller explanation and illustration, see Wesley Salmon's *The Foundations of Scientific Inference* (hereafter '*Foundations*') (The University of Pittsburgh Press, Pittsburgh, 1966), pp. 56–65.

[16] This is an *absolute* sense of 'confirms'. We can also use that term in a *relative* sense, and say that A confirms B with respect to C when $P(B/(A \& C)) > P(B/C)$. In this sense A confirms B (relative to background information C) when the probability of B on A together with background information exceeds that of B on the background information alone.

[17] See de Finetti's essay in *Studies in Subjective Probability*, H. Kyburg, Jr. and H. Smokler (eds.) (Wiley, New York, 1964).

[18] *The Logic of Decision* (McGraw-Hill Co., New York, 1965).

[19] 'Slightly more Realistic Personal Probability', *Phil. Sci.* 34 (1967).

[20] A point remarkably anticipated in the immortal 'The Problem with the Inductive Argument from Evil on the Personalist Interpretation of the Probability Calculus' (variously attributed to Dietricht of Dordrecht (1494–1581) and Lucien of Marseille (1550–1632):

> The Atheological claim
> Enjoys moderate but ill-deserved fame
> For on the personalist interpretation
> It's mere biographical information
> And thus is polemically lame.

[21] See Salmon, *Foundations*, pp. 79–82.
[22] See, for example, the 'Preface to the Second Edition' of his *Logical Foundations of Probability* (Univ. of Chicago Press, Chicago, 1962).
[23] *An Introduction to Confirmation Theory* (Methuen & Co., Ltd., London, 1973), p. 99.
[24] And if we think there could be infinitely many horses, we can add H_∞, a proposition to that effect, to *H*.
[25] Mary Hesse, for example; see her *The Structure of Scientific Inference* (Univ. of California Press, Los Angeles, 1974), Ch. 8.
[26] Carnap, *The Logical Foundations of Probability* (Univ. of Chicago Press, Chicago, 1950 (second edition 1963)), pp. 570–577.
[27] 'Towards a Theory of Inductive Logic', in Y. Bar-Hillel (ed.), *Logic, Methodology and Philosophy of Science* (North-Holland Publ. Co., Amsterdam, 1965), pp. 274–289, and 'A Two-Dimensional Continuum of Inductive Methods', in J. Hintikka and P. Suppes (eds.), *Aspects of Inductive Logic* (North-Holland Publ. Co., Amsterdam, 1966), pp. 113–133.
[28] *The Theory of Probability* (The University of California Press, Berkely, Calif., 1949).
[29] These considerations have led to a *propensity interpretation* of probability, championed by Karl Popper and others. See 'The Propensity Interpretation of the Calculus of Probability and the Quantum Theory', in S. Korner (ed.), *Observation and Interpretation: A Symposium of Philosophers and Physicists* (Butterworth, London, 1957), (2nd ed. entitled *Observation and Interpretation in the Philosophy of Physics*).
[30] Salmon, *Foundations*, p. 83.
[31] See David Lewis, *Counterfactuals* (Harvard Univ. Press, Cambridge, Mass., 1973), pp. 79–80.
[32] *Dialogues Concerning Natural Religion*, Part II (I assume Philo speaks for Hume).
[33] Salmon follows Reichenbach in writing '$P(A, B)$' for the probability from A to B – that is, for the probability of B given A. In order to avoid confusion, I shall rewrite his formulas in the style to which we have become accustomed, writing '$P(A/B)$' for the probability of A given B and replacing his set theoretical symbol '$\cap$' by the propositional symbol '&' for conjunction.
[34] *NN*, p. 192.
[35] See, for example, Walter Kaufmann's *Critique of Religion and Theology* (Harper Brothers, New York, 1958), pp. 72–96, Antony Flew, *The Presumption of Atheism* (Harper & Row Publishers, Inc., New York, 1976), p. 15, and many others.
[36] *Encyclopedia of Sacred Theology*, tr. by J. Hendrick de Vries (Charles Scribner's Sons, New York, 1898) p. 143.
[37] And perhaps some parts of *B* will be thought themselves to require a foundation in other propositions. According to the classical (Aristotelian and Thomistic) version of foundationalism only self-evident propositions (propositions like 2 + 1 = 3) and propositions evident to the senses (ones like *that tree has no leaves*) belong in the foundations of a noetic structure. According to modern or Cartesian foundationalism, the propositions properly in the foundations of a noetic structure are those that are self-evident or, like *I seem to see a tree*, incorrigible. See my paper 'Is it Rational to Believe in God?', in *Rationality and Religious Belief*, Neil Delaney (ed.) (Univ. of Notre Dame Press, South Bend, 1978).
[38] For a partial and preliminary attempt to discuss some of them, see 'Is It Rational to Believe in God?', *loc. cit.*