BOB AND CAROL AND TED AND ALICE

1. THE PROBLEM

Consider the following:

(1) The last word of (1) is obscene.
(2) The last word of (1) is obscene.

It would appear that (1) cannot be turned into a truth by addition of quotation marks, but that (2) can be so changed – namely, by putting quotation marks around its last word. Yet it would also appear that (1) = (2); and if this is so, then by Leibniz' Law whatever is true of (2) is also true of (1). How is this apparent contradiction to be resolved?[1]

2. PRELIMINARIES

Call the sentence token which occurs in the line indexed above by '(1)', 'Bob'. Call the sentence token which occurs in the line indexed by '(2)', 'Carol'. Bob and Carol are twins. Using '*T*' to abbreviate 'the type of', we can express this as follows:

$$\text{Bob} \neq \text{Carol, but } T(\text{Bob}) = T(\text{Carol}).$$

Suppose that next Sunday morning I add quotation marks to Carol's last word (token), and the following Monday morning I do the same to Bob. By the following Tuesday morning, they would both look like this:

The last word of (1) is 'obscene'.

Call Bob's descendent 'Ted', and Carol's descendent 'Alice'. Ted and Alice are also twins.

$$\text{Ted} \neq \text{Alice, but } T(\text{Ted}) = T(\text{Alice}).$$

In order to decide whether Bob and Carol and Ted and Alice are true we must know to whom they are referring. Clearly the Great Designer,

Hintikka et al. (eds.), Approaches to Natural Language, 490–518.

Professor Cartwright, designed them to use '(1)' to refer to the individual dubbed '(1)' by the '(1)' which occurs to the left of Bob. Call that token of '(1)', 'Index-1'. *Our* use of '(1)' is governed by the use of Index-1. A point of the problem is to make *their* use of '(1)' co-referential with ours.[2]

Index-1 occurs as part of an act of dubbing in which what is displayed to the right of Index-1 is dubbed '(1)'. Our dubbings, of Bob and Carol and Ted and Alice and Index-1, have all been by description – "Call the blah blah blah, 'Bob'." But the dubbing which occurs in the line containing Index-1 is a dubbing by demonstration – "Call this: ___ '(1)'." So, Index-1 must refer to whatever is displayed to its right.

Bob is certainly displayed there, but it seems equally appropriate to claim that T(Bob) is displayed there.

3. THE OBVIOUS SOLUTION

For this solution we assume that it is always a sentence *type* that is displayed in dubbings of the kind in question. Thus:

$$(1) = T(\text{Bob}) = T(\text{Carol}) = (2).$$

Is it true that in violation of Leibniz' Law (2) can be changed into a truth by the addition of quotation marks but (1) cannot?

Let us begin by discussing (2) in both its actual form, T(Carol), and its potential form, T(Alice). T(Carol) is not true[3] because the last word of (1), namely, the word 'obscene', is not itself obscene. However, T(Alice) is true (allowing for a tacit shift from the 'is' of predication to the 'is' of identity) because the last word of (1) is the word 'obscene'.

But wait a minute! On Sunday morning, when Alice first appears, she (or, if you prefer, her type) is true. However by Monday afternoon, when Ted has replaced Bob, the last word of (1), i.e., the last word of the referent of Index-1, seems to be the word ' 'obscene' '. Thus at that time Alice degenerates to falsity.

Alice's apparent instability is illusory. On Monday morning, when we replace Bob with Ted, we replace the display in a dubbing. Since we neglect to simultaneously replace the name being bestowed, distinct entities are given the same name. Horrors! There is the old (1), T(Bob); and there is (1) Jr., T(Ted).

If at her birth on Sunday, Alice uses '(1)' to refer to T(Bob), then there is no reason to believe that Bob's replacement by Ted should cause her to forget Bob and begin using '(1)' to refer to *T*(Ted). Indeed, her type and that of her mother are both named '(2)', but this has not caused her to forget her own mother, nor to confuse their differently truth valued types. Alice may continue to refer to whichever (1) she referred to on Sunday. This allows that she may – presciently – have referred to *T*(Ted) all along.

Alice's constancy aside, the conclusion is that so long as the twins refer to the same (1) they have the same truth value.[4]

When (2) is changed, it is changed into a truth with respect to (1), but a falsehood with respect to (1) Jr. *Exactly the same holds when* (1) *is changed!* Thus Leibniz' Law applies without contradiction.

The puzzle was generated by thinking that both (2) and (2) Jr. must refer to (1); whereas both (1) and (1) Jr. must be self-referential. Thus (2) Jr. and (1) Jr. would refer to different sentences. The puzzle is resolved by recognizing that there are two (1)'s and keeping track of which (1) is under discussion.

4. A MORE INTERESTING SOLUTION

There is a grave difficulty in the obvious solution. The problem speaks the language of 'turn into' and 'change into', but the solution is couched in a metaphysics of replacement.[5]

We did not *change* the false Carol into the true Alice, we *replaced* the false Carol with the true Alice. Or did we? What really happens when I take my pen to Carol next Sunday morning? Could it be that Alice and Carol, like Hesperus and Phosphorus, are one?

There is every reason to think so. Sentence tokens are physical objects and macro-objects at that. They are created, wear down, fade, are touched up, and sometimes are distorted. Neon sentence tokens frequently malfunction and thereby change type. If sufficiently comical, such transformations are enshrined in *The Reader's Digest*.

I conclude that:

Carol = Alice and Bob = Ted.

This not only accounts for the critical use of 'changed' in the formulation of the problem, but as we shall see, it also illuminates the respect in which Carol *can* be changed into a truth by the addition of quotation marks while Bob cannot.

Our bookkeeping simplifies. We can make the natural assumption that only two tokens are involved, Bob-Ted and Carol-Alice, and also that only two dubbings are involved, one incorporating Index-1 and one incorporating its colleague, Index-2. Index-1 stands beside the same token, Bob-Ted, throughout the period of interest. In the problem it is (1) and (2) that are 'changed'. So it must be intended that:

(1) = Bob-Ted, and (2) = Carol-Alice.

With only one (1) to contend with, we can make the natural assumption that throughout the period of interest both Bob-Ted and Carol-Alice use '(1)' to refer to Bob-Ted.

Both (1) and (2) are false at the present time. But their potentialities differ.

(2) *can* be transformed into a truth by putting quotation marks around her last word. In fact, next Sunday morning she *will* be so transformed. Note that this possibility depends on the possibility of making no earlier transformation in (1). When quotation marks are put around the last word in (1), on Monday morning, (2) will again change in truth value. This time not because *she* has changed, but because the world has changed around her and she has viewed it as unchanged.

In contrast, (1) *cannot* be transformed into a truth simply by the addition of quotation marks to his last word. In particular, when those quotation marks are added next Monday morning, his revised self-analysis is true only of his unrevised self. Thus he continues to dissemble. In order to change (1) into a truth, a second change must be made so that (1) looks like this:

The last word in (1) was 'obscene'.

5. A COMPLETE SOLUTION

The preceding solution, though it adequately accounts for the critical elements of *change* and *self-reference*, is yet only a partial solution to the original problem. A complete solution must, in addition, satisfy all three of the following paradoxical conditions:

(1) cannot be changed into a truth by addition of quotation marks to its last word,

(2) can be changed into a truth by addition of quotation marks to its last word,

(1) = (2).

According to the preceding solution, (1) = Bob-Ted ≠ Carol-Alice = (2). Thus the preceding solution clearly fails to satisfy the third condition. This is a cheap avoidance of paradox, no more subtle in this respect than the obvious solution, which simply fails to satisfy the first condition.

In order to obtain a complete solution we must abandon our preliminary claim that Index-1 is used to dub some individual displayed to its right. In a dubbing, a proper name is introduced. But treating Index-1 as a proper name, whether of Bob-Ted or Bob-Ted's current type, is what led to the incompleteness of the previous solutions.

Thus, what is required is an analysis which treats Index-1 as semantically complex. Index-1 must refer to a type, but not by naming it as in the obvious solution. Instead Index-1 should be thought of as *describing* its referent, in the manner of the functional expression, 'the type of *this*'. The only *naming* involved is that of the component demonstrative 'this', which names what is displayed – in the present case, the token Bob-Ted. Since we never replace the display, the demonstrative 'this' always refers to Bob-Ted. If we assume that a proper name functions rather like a demonstrative with a fixed demonstratum, we might describe Index-1 as semantically equivalent to '*T*(Bob-Ted)'. When Bob-Ted changes, '*T*(Bob-Ted)' takes on a new referent.

The treatment of Index-2 clearly should parallel that of Index-1. We can express the strong equivalence of '*T*(Bob-Ted)' with the use of '(1)' introduced by Index-1, and of '*T*(Carol-Alice)' with the use of '(2)' introduced by Index-2, roughly as follows:

(a) Necessarily ((1) = *T*(Bob-Ted)), and
necessarily ((2) = *T*(Carol-Alice)).

If the third condition on a complete solution is to be satisfied, Index-1 and Index-2 must refer to types as in the obvious solution. But if the first two conditions are to be satisfied, Index-1 must reflect the self-referential element represented in the more interesting solution. The present treatment is simply the natural way to combine the advantages of each of the previous solutions.

Given this interpretation of '(1)', how shall we treat the predicate "can be changed into a truth by addition of quotation marks"? This too has a simple and natural interpretation.

Consider first an analogous predicate. Let *M* be a metal bar exactly one meter long. A typical claim for a potential of change is:

(b) *M*'s length can be changed to more than a meter by heating it to 200°.

Change is mentioned, and change is indeed involved. But a change in *M*, not a change in the length: *one meter*. *M*, not *M*'s length, is heated; as a consequence, *M*'s former length, one meter, is replaced by a new length, 1.001 meters. Ignoring the subtleties involved in the use of 'can' as opposed to 'would', and also ignoring the presupposition that *M*'s length is not now more than one meter, an approximate equivalent to (b) is:

(c) If *M* were heated to 200°, then *M*'s length would be more than one meter.

The purpose of this example is to point out the *intensional context* involved in (b).[6]

Returning to the present interpretation of '(1)', we expand the first condition for a complete solution in the style of (c):

(d) It is not the case that, if quotation marks were put around Bob-Ted's last word, then *T*(Bob-Ted) would be true.

To establish that (d) holds, suppose that quotation marks *were* put around Bob-Ted's last word. Bob-Ted would then look like this:

The last word of (1) is 'obscene'.

Recalling that it is an assumption of the problem that both Bob-Ted and Carol-Alice always use '(1)' as we do, we see, by (a), that *T*(Bob-Ted) would then be true if and only if the last word of what would then be *T*(Bob-Ted) were the word 'obscene'. But the last word of what would then be *T*(Bob-Ted) would be ' 'obscene' ' not 'obscene'. Hence *T*(Bob-Ted) would not be true. Hence the subjunctive conditional in (d) does not hold. Hence (d), and thereby the first condition for a complete solution, is satisfied.

The second condition for a complete solution expands as follows:

(e) If quotation marks were put around Carol-Alice's last word, then *T*(Carol-Alice) would be true.

Arguing as above, we see that (e) is satisfied if and only if the placing of quotation marks around Carol-Alice's last word would leave the last word of *T*(Bob-Ted) (currently the word 'obscene') unaffected. Since the stability of Bob-Ted surely *is* one of the background conditions to be assumed in evaluating a subjunctive conditional like (e), it follows that (e), and thereby the second condition for a complete solution, is satisfied.

Bob-Ted and Carol-Alice *currently* have the same type. Thus, by (a), the third condition is also satisfied.

Our solution is therefore complete.

APPENDIX I: THE ADDITION OF QUOTATION MARKS

In the preliminaries, quotation marks were added directly to the token Bob, and *T*(Ted) was taken to be the type so tokened. An alternative is to treat the addition of quotation marks as an operation applied directly to the type *T*(Bob), and yielding the type *T*(Ted).

Homework Problem # 1. The alternative treatment leads to a solution even less interesting than the obvious solution. What is it?

Homework Problem # 2. Can the three solutions given above be reconstructed using the alternative treatment of quotation marks?

APPENDIX II: TYPES, TOKENS, AND REFERENCE

Although in the obvious solution *T*(Bob) = *T*(Carol), it did not immediately follow that Bob and Carol share a truth value. Tokens of 'Ari is so clever' in the mouths of Plato and Jackie could differ in truth value. Tokens of 'I am so clever' in the mouths of Plato's Aristotle and Jackie's Aristotle could differ in truth value.

Homework Problem # 3. Do the two pairs of twins (of the types 'Ari is so clever' and 'I am so clever') differ in the same way?

APPENDIX III: A NONSOLUTION

It might be thought that the original problem could be dissolved simply by claiming that (1) = Bob and (2) = Carol. Then (1) $\neq$ (2). Hence no application of Leibniz' Law is possible. Hence no paradox. But this leaves unexplained how twins can differ in truth value when they do not differ in the ways discussed in Appendix II. The use of twins to construct the puzzle is, in fact, inessential.

Homework Problem # 4. Reconstruct the original problem and discuss its solution using the following:

(Dick) My last word is obscene.
(Helen) Your last word is obscene.

APPENDIX IV: TRUTH AND CONTENT

It may be thought that another plausible candidate for the referent of Index-1 is the *content* of Bob – the proposition expressed by *T*(Bob) in the context in which Bob occurs. Indeed, the problem uses language of the form:

(1) is not true.

How can truth or falsity be predicated directly of either a token or a type? (1) must be a proposition. But the same proposition is expressed by each of the following:

The last word of (1) is obscene.
An obscene word is the last word of (1).

So if (1) is a proposition, how can the function *the last word of* be applied directly to (1)?

To make sense of the conditions of the problem, both of the following must be meaningful:

(i) the last word of (1)
(ii) (1) is false.

We have chosen to interpret '(1)' in such a way that (i) has an obvious meaning. (ii) is then accommodated by implicit (and sometimes, explicit) relativization to features which fix the content of a fugitive sentence. Among the features implicitly taken into account are that the language is English. Among the features explicitly accounted for are the referent of the '(1)' contained in (1) (see note 3). In the obvious solution we spoke of (1)

Jr. being *true with respect to* (1) but *false with respect to* (1) Jr. Similarly, in the more interesting solution when the tense of (1) became relevant, the notion of truth used was that of (1) being *true on Tuesday morning*.

Homework Problem # 5. Construct a solution in which the referent of '(1)' is such that truth is not relativized as above. That is, construct a solution in which the content is built into (1).

APPENDIX V: THE INDIVIDUATION OF TYPES

I have suggested that the most natural notion of a token allows a token to change its type – in the sense that a token can be so changed that a new type will replace its former type. What principle of individuation should we use for types? It is not really necessary that homographous words should share a type. If a useful notion of type can grant the tokens:

homographous
homographous

the same type, why should it deny 'yellow' (a color) and 'yellow' (a character) distinct types?

Homework Problem # 6. Do the verb 'paint' and the noun 'paint' have distinct types?

APPENDIX VI: CONGRUENCE AND IDENTITY

We might have said that although Bob-Ted ≠ Carol-Alice, there are times at which Bob-Ted *is congruent with* Carol-Alice. We could have symbolized this with an explicit three-place predicate:

Cong(Bob-Ted, Carol-Alice, *t*)

or with a tensed two-place predicate:

Bob-Ted ≈ Carol-Alice
Next Sunday morning (Bob-Ted ≉ Carol-Alice)

where 'next Sunday morning' is a temporal operator treated in the standard way.

Instead, in order to achieve a real identity between (1) and (2), we introduced a tensed functor: '*T*'. Thus '*T*(Bob-Ted) = *T*(Carol-Alice)', with tenseless '=', is true at the same times as 'Bob-Ted ≈ Carol-Alice'.

Homework Problem # 7. Under what conditions on the three-place congruence relation can the tensed predicate '≈' be traded off for a tensed functor and real (i.e., tenseless) identity?

APPENDIX VII: WHAT CAN BE DISPLAYED?

A dubbing by demonstration takes the form:

Let us call this: _ _ _ 'McBlank'.

A dubbing by description takes the form:

Let us call α 'McAlpha',

where the blank is replaced by the individual being dubbed, and 'α' is replaced by a description of the individual being dubbed.

It would be good if dubbings by demonstration and dubbings by description were to correspond respectively to dubbings with the subject present and dubbings in absentia.[7] But first some problems concerning display potentials must be resolved.

Some individuals, like the universe, are hard to display all at one place because they are difficult to gather up. Some individuals, like Quine, are hard to display all at once because, as he would protest, "of my hence and ago." Other individuals, like 'Quine' and red are hard to display because they themselves are not within space-time, though their manifestations are. Still other individuals, like nine and the null set, neither are, nor have manifestations, within space-time.

Nine and null can probably only be dubbed by description. But things like Quine, 'Quine', red, and the universe, which have locally presentable aspects or manifestations might be deemed demonstrable in themselves.

There are epistemological reasons for coming to think, as Russell did, that only completely local beings can be demonstrated directly. On this view when I point to Venus and say 'this planet', I am giving a *description* of Venus which incorporates a *demonstration* of one aspect of Venus. Such a treatment provides a Fregean explanation of how a long slow utterance of:

This planet [pointing to Venus in the morning] = this planet [pointing to Venus in the evening]

can be both informative and true. The denoting phrases are thought of as stylistic variants of 'the planet of which *this* is an aspect'.

On the other hand it seems more natural to think of nice solid continuous four dimensional objects as typical of the kind of thing we point at (directly), and to think of their aspects and stages as somehow derived and abstracted (by description).

Homework Problem # 8. Can Quine be demonstrated or only described?

Homework Problem # 9. Are Quine's aspects and stages like 'Quine's manifestations'?

Homework Problem # 10. Are 'Quine's manifestations' like red's?

Homework Problem # 11. How do we dub nine and null?

Only on a view such as Russell's is it at all reasonable to make it a prerequisite for a dubbing that the dubbor *know*, or stand in some other special epistemological relationship to, the dubbee. Though most pointings are *teleological* (the finger is aimed at a preconceived individual), *blind demonstrations* (as in spin-the-bottle) are also possible and provide an equally satisfactory basis for a dubbing. Descriptions also may be either teleological or blind. A description like 'the first child to be born in the twenty-second century' is near-blind.

Homework Problem # 12. How much was known of Jack the Ripper when he was so dubbed?

APPENDIX VIII: THE AMBIGUITY OF DEMONSTRATIONS

There are conventions governing what is demonstrated when I point. I cannot aim my finger at you and thereby refer to myself. Even though you and the rest of my auditors know that I have mistaken you for your twin, I cannot aim my finger at you and thereby refer to your twin. But in cases like that of Index-1 and cases where my finger is genuinely aimed at a boy, his jacket, and its zipper the conventions are not completely determinative. The only further resource available to resolve the issue seems to be my intentions, taken in a broad sense to include that which guided my pointing. If we wish to avoid introducing an intentional element into the truth conditions for assertions in which 'this' is completed by a pointing, we might require that 'this' always be accompanied by a common noun phrase – 'this boy', 'this zipper', 'this momentary stage of a rabbit surface'. When my finger aims at more (or less) than one such, the demonstrative phrase could be treated in the manner of an improper description. The more general commoun noun phrases, 'physical object', 'entity', would invariably produce improper demonstrations.

Homework Problem # 13. If one points at the center of a pool of blood, is the demonstrative phrase 'this blood' proper or improper?

Homework Problem # 14. Does the correct solution to the problem – and in particular to the question of what is displayed to the right of Index-1 – depend on what Cartwright had in mind?

Homework Problem # 15. Donnellan's account (1966, 1968, 1970) of the referential use of a description is more along intentional lines. If he were to adapt his account to pointings, what would he say about the mistaken pointing at a twin?

APPENDIX IX: RIGID DESIGNATORS

The introduction of an expression which is a simple name syntactically, but a compound description semantically, I call an *abbreviation* – to contrast with the more common form of introduction, a *dubbing*. Proper names are, or at least purport to have been, introduced by dubbings. Since the introduction of a syntactically simple expression, like Index-1, is almost invariably a dubbing, I took special care to point out that in the complete solution I was interpreting the introduction of Index-1 as an abbreviation.

The semantical differences between descriptions like 'the number of planets' and proper names like '9' are already familiar. The description may denote different numbers under different circumstances, but the name always denotes the same number. It has been less widely noticed that in this respect all proper names are like '9'. In fact, the very purpose of introducing a proper name is often to provide an expression free from the vagaries of 'the number of planets'. Kripke (1972) has remarked that proper names are *rigid designators* – the same name designates the same individual in all circumstances. I add that the introduction of a proper name may as well be occasioned by frustration over the flaccidity of a description as by frustration over its length. Discussion of an individual's potentiality to fail to fulfill the description by which he is known, will almost always be facilitated by the introduction of a proper name. The yacht owner's guest who is reported by Russell to have become entangled in "I thought that your yacht was longer than it is" should have said, "Look, let's call the length of your yacht a 'russell'. What I was trying to say is that I thought that your yacht was longer than a russell." If the result of such a dubbing were the introduction of 'russell' as a mere abbreviation for 'the length of your yacht', the whole performance would have been in vain.

Through its use in a dubbing by description, an arbitrary description can produce a name which *rigidly* designates whatever the description *happens* to describe in the context of the dubbing.

Homework Problem # 16 (adopted from Kripke). '100° Centigrade' is *defined* as 'the temperature at which water boils at sea level'. Are such definitions dubbings or abbreviations?

Homework Problem # 17. The insertion of words like 'present' and 'actual' in a description – 'the *present* Queen of England', 'the *actual* length of your yacht' – cause the description to take the referent it would have if it were not within the scope of any temporal, modal, epistemological, or other intensional operators. In Russell's language, they give the description *primary scope.* Thus the insertion of such words fixes the referent independently of any intensional operators within whose scope the description lies. Do such words convert the description into a rigid designator?

Others, before Kripke, had recognized the rigidity of proper names. His notable contribution has been to indicate a technique for *finding* the referent of a proper name, on a particular occasion of use, which is independent of the knowledge and belief of the user. The technique consists in tracing the history of acquisition of the name from use back to bestowal. It is based on the exceedingly plausible assumption that if a name enters your vocabulary from hearing me use it (you learn the name from me), then your utterances of the name have the same referent as mine. Kripke's technique for finding the referent frees proper names from their supposed dependence on currently associated descriptions[8] and thus eases the way for recognition of their rigidity.

I have attempted to supplement the view by emphasizing the techniques for bestowing a proper name and thus *fixing* reference. I call such acts of bestowal 'dubbings'. (Other terms are available, but they tend to carry a sectarian bias.) The resulting view of the reference of proper names can be encapsulated as follows:

If α is the proper name used on some particular occasion, then
(i) α denotes x iff α originated in a dubbing of x, and
(ii) for all possible circumstances w, α denotes x with respect to w iff α denotes x.

It is a corollary that if α did not originate in a successful dubbing (one which is a dubbing of *some* x), α nowhere denotes anything.

This view of the reference of proper names is anti-intentional. It says what the *name* (in use) refers to, not what a *user* refers to, or intends to refer to, or is most plausibly taken to be talking about, in *using* the name. The latter (user's reference) is an important, but different, sense of 'refer'. Suppose the name 'Jaakko Hintikka' is introduced to me by having Julius Moravcsik introduced to me with the lie "This is Jaakko Hintikka." When I later remark, "Hintikka's Finnish accent is a very unusual one," I, no doubt, am talking about Moravcsik. I may even be said to have referred to him. But my *utterance* of the name refers to Hintikka. Thus the sentence token I have uttered is false. (There may be other Hintikka's with unusual Finnish accents, but the Finnish accent of the Hintikka referred to in the lie is usual. Remember it was a lie, so the 'this' and the 'Jaakko Hintikka' could not be co-referential.) I see no way, other than speaking carefully, of avoiding the ambiguating effects of this distasteful dualism.

Homework Problem # 18. Kaplan (1968, especially §IX) has introduced a peculiar relation between an occurrence of a name and an individual, which he expresses with an italicized '*of*'. To which of the following does his notion correspond: the name's reference, the user's reference, some confused combination of the two, none of the above?

APPENDIX X: DENOTATION AND EXISTENCE

Some have claimed that though a proper name might denote the same individual with respect to any possible world (or, more generally, possible circumstance) in which he exists, it certainly cannot denote him with respect to a possible world in which he does not exist. With respect to such a world there must be a gap in the name's designation, it designates nothing. This is a mistake.[9] There are worlds in which Quine does not exist. It does not follow that there are worlds with respect to which 'Quine' does not denote. What follows is that with respect to such a world 'Quine' denotes something which does not exist in that world. Indeed, Aristotle no longer exists, but 'Aristotle' continues to denote (him).

The view that no expression could name Quine with respect to a possible world in which he does not exist seems to be based on one of two ideas. The first is usually expressed with respect to possible worlds, but I will caricature it with respect to the moments of time.

Individuals are taken to be specific to their moment, thus they are momentary stages of what *we* would call individuals. Variables and constants, when evaluated with respect to a moment t, take as values stages occurrent at t. *Our* individuals can be constructed from these individuals (which were sliced out of our individuals in the first place) by assembly (or, perhaps, reassembly). The assemblages of stages are used to evaluate quantification into and out of temporal operators. Although you cannot literally step in the same river twice, you can step in two stages of the same assemblage. A variable which recurs within and without a temporal operator will take different values in its different occurrences, but its values will be from the same assemblage.[10] Note that though each stage belongs to one or more assemblages, the values of the variables are not assemblages but stages. The individuals are stages. Genidentity, as determined by the assemblages, holds between distinct stages.[11]

Homework Problem # 19. Let **T** be the set of moments of time ordered by $<$. The present time is 0. Let $\mathbf{S}(t)$, for $t \in \mathbf{T}$, be the set of stages occurrent at t; let $\mathbf{F}(t)$, for $t \in \mathbf{T}$, be the subset of $\mathbf{S}(t)$ of which 'F' is true at t; let **A** be the set of assemblages f, where the domain of f is included in T and for each t in the domain of f, $f(t) \in \mathbf{S}(T)$. The operator 'P' is read 'at some earlier time'. Translate the following sentence, involving a quantification out of a temporal operator, into the metalanguage:

$$P[\forall x(Fx] \rightarrow Fx)$$

(In English: There is a certain time in the past such that all individuals, of that time, who were then female still are.)[12]

According to the foregoing view, at each moment of his lifetime 'Aristotle' denoted a different entity, the Aristotle of the moment. Thus, at the present moment, when no current entity is sufficiently well connected to the other Aristotle stages to be an Aristotle stage, 'Aristotle' denotes nothing. What should it denote, a stage of Quine?[13] But according to this view, there is no real Aristotle to be denoted, only the Aristotles of each moment, so this view, in its pure form, is too bizarre to support the mistake.

A compromise is proposed. Continue to think of things as before, but take the assemblages themselves as the values of the variables and constants. Whenever a term denoted a stage, let it now denote that stage's assemblage (or one of them). Whenever a term denoted nothing (i.e., at those times not in the domain of a relevant assemblage), let it still denote nothing. Here is the mistake in full bloom.

The original view may have been bizarre, but it had its uses in explicating bizarre notions, for example that I might change into twins or that twins might have changed into me.[14] The compromise view does not have one becoming two, instead it has two coincident assemblages diverging. An unusual situation, but one not violative of Leibniz' Law. As individuals,

assemblages are quite well behaved. Thus no reason remains not to take them as values of their proper names with respect to moments when they do not exist.[15] If, on the compromise, 'Quine' denotes the same thing yesterday and today, why not let 'Aristotle' denote the same thing 2300 years ago and today? After all, it does.

The second idea that might lead one to doubt that 'Quine' could denote where Quine does not exist is a simple confusion between our language and theirs. For reasons to be adumbrated shortly, ever-unactualized possibilia are extraordinarily difficult to dub. Thus the inhabitants of a world in which Quine never exists would likely have no name for him.[16] So what! He exists here. *We* have a name for him, namely, 'Quine'. It is *our* terms and formulas whose denotation and truth value are being assessed with respect to the possible world in question.

Homework Problem # 20. If a horse's tail were called a 'leg', horses would have five appendages called 'legs'. How many legs would a horse have?

Homework Problem # 21. Does 'Quine' denote Quine with respect to the time of Aristotle's birth? Who was then called 'Quine'?

APPENDIX XI: NAMES FROM FICTION

I have argued that 'Aristotle' denotes something which, at the present time, does not exist. I could now argue that 'Pegasus' denotes something which, in the actual world, does not exist. I shall not. Pegasus does not exist, and 'Pegasus' does not denote. Not here; not anywhere. What makes 'Aristotle' more perfect than 'Pegasus'?

The 'Aristotle' we most commonly use originated in a dubbing of someone,[17] our 'Pegasus' did not. Some rascal just *made up* the name 'Pegasus',[18] and he then pretended, in what he told us, that the name really referred to something. But it did not. Maybe he even told us a story about how this so-called Pegasus was dubbed 'Pegasus'. But it was not true.

Maybe he proceeded as follows. First, he made up his story in Ramsified form: as a single, existentially quantified sentence with the made up proper names ('Pegasus', 'Bellerophon', 'Chimaera', etc.) replaced by variables bound to the prefixed existential quantifiers; second, he realized that the result was possible, and that therefore it held in some possible world, and that therefore there was at least one possible individual who played the winged horse in at least one possible world; and third, he tried to dub one of those possible individuals 'Pegasus'. But he would not succeed. How would he pick out just one of the millions of such possible individuals?

Homework Problem # 22. Suppose that Quine and Kripke both might have been winged horses of the kind described in the story. Which one, if either, is Pegasus? (Hint: remember that 'Pegasus' is a rigid designator, so whoever might be Pegasus *is* Pegasus.)

I do not assume that there are no proper names which succeed in naming ever-unactualized possibilia (be they individuals, worlds, or circumstances). But the dubbing problem raises serious questions about the content of discourse using such putative proper names. I fear that those who would so speak have adopted the logician's *existential instantiation* as a form of dubbing:

> There is at least one cow in yonder barn. Let's call one of them 'Bossie'. Now, how much do you think she weighs?

I am skeptical of such dubbings. The logician is very cautious in *his* use of the names so derived.[19]

The requirement for a successful dubbing is not that the dubbor know who the dubbee is. As remarked in Appendix VII, the dubbor can point with his eyes closed or use a description like 'the first child to be born in the twenty-second century'. The requirement is simply that the dubbee be, somehow, uniquely specified. This our story teller has not succeeded in doing. Probably he did not even try.

Perhaps I am being too harsh on 'Pegasus'. I have treated a myth as if it were pseudo-science, and dismissed it for failure of factuality. Even pseudo-science may have something to offer other than factuality.

Suppose we start out by acknowledging that the Pegasus-myth is FICTION.[20] Still it is, in a sense, possible. Should we not take 'Pegasus' to denote what it denotes in *the world of the myth*? We must be very careful now.

If 'the world of the myth' is meant to refer to the (or even, *a*) possible world with respect to which the myth – taken as pseudo-science – is true, there is an immediate objection. As given, the myth uses the name 'Pegasus'. Thus its truth with respect to a possible world requires a *prior* determination of what, if anything, 'Pegasus' names with respect to the possible world. Suppose we turn, then, to the Ramsified myth. Although it will be true in millions of possible worlds, Ramsification eliminates the very name whose denotatum we seek.

An alternative strategy arises in connection with the Ramsified myth. Wherever it is true, *something* plays Pegasus. If we limit attention to those cases where exactly one thing plays Pegasus, we can refer to it by means of the description ⌜the x $\mathscr{M}$⌝, where $\mathscr{M}$ is the Ramsified myth without the existential quantifier which binds the variable 'x' which replaced all occurrences of 'Pegasus' in the myth as given. Why not take 'Pegasus' to *abbreviate* ⌜the x $\mathscr{M}$⌝?[21] The objection to this wonderfully candid proposal is that the Friend of Fiction is unlikely to accept it. First, 'Pegasus' loses the status which allowed it to function so smoothly in 'Bellerophon hoped that Pegasus...' contexts. The expansion of such declarations is awkward at best. Second, there is no fixed individual, Pegasus, denoted by 'Pegasus' with respect to all possible worlds in which he exists. Third, 'Pegasus' still denotes nothing. When the presumed dubbing is disregarded and 'Pegasus' ceases to be a rigid designator, the world of the myth ceases to be of interest.

There is another interpretation of 'the world of the myth' which, I believe, better represents the position of those who take the view that 'Pegasus' finds its denotatum in the world of the myth.[22] The myth is possible in the sense that there is a possible world in which it is truthfully *told*. Furthermore, there are such worlds in which the language, with the exception of the proper names in question, is semantically and syntactically identical with our own. Let us call such possible worlds of the myth, '*M* worlds'. In each *M* world, the name 'Pegasus' will have originated in a dubbing of a winged horse. The Friend of Fiction, who would not have anyone believe the myth (even Ramsified), but yet talks of Pegasus, pretends to be in an *M* world and speaks its language.

But beware the confusion of our language with theirs! If w is an *M* world, then *their* name 'Pegasus' will denote something with respect to w, and *our* description 'the x such that x is called 'Pegasus'' will denote the same thing with respect to w, but *our* name 'Pegasus' will still denote nothing with respect to w. Also, in different *M* worlds, different possible individuals may have been dubbed 'Pegasus'; to put it another way, *our* description 'the x such that x is called 'Pegasus'' may denote different possible individuals with respect to different *M* worlds.

I do not object to the inhabitants of one of the *M* worlds remarking that their name 'Pegasus' denotes something with respect to *our* world that does not exist in our world. But I reserve the right to retort that *our* name 'Pegasus' does not even denote with respect to their world.

To summarize. It has been thought that proper names like 'Pegasus' and 'Hamlet' were like 'Aristotle' and 'Newman 1', except that the individuals denoted by the former were more remote. But regarded as names of *our* language – introduced by successful or unsuccessful dubbings, or just made up – the latter denote and the former do not.

Homework Problem # 23. Is the foregoing account of proper names deriving from fiction correct? If so, how could its fourth sentence be true?

APPENDIX XII: THE UNIVERSE OF DISCOURSE

At the present time, the techniques are available to produce a completely axiomatized formal theory of definite descriptions to fit almost any specification. We should now more carefully distinguish that part of the metalinguistic apparatus which consists of logicians' tricks, adopted for purely instrumental reasons and devoid of philosophical import, from that part which directly realizes the intended interpretation of the object language.

It may be technically convenient to introduce an entity, †, completely alien to the universe of discourse of the object language and to adjust slightly our use of 'denotes' so that we can say that a singular term α does *not* denote, in the following odd way:

α so-to-speak-denotes †.

We have not lost sight of the fact that α does not really denote, *denotation* and *so-to-speak-denotation* are interdefinable. The use of the latter is fairly described as a logician's trick for smoothing some definitions in the metalanguage. Though it seems unlikely, it may even turn out to be useful to introduce more than one such way of saying that α does not denote.

Definite descriptions are rather special kinds of terms. A definite description ⌜the x ϕ⌝ is proper if among the values of 'x' there is a unique individual satisfying ϕ. As ordinarily conceived, a proper definite description denotes one of the values of the variables, and an improper definite description does not denote at all (though of course it may so-to-speak-denote something). Thus a definite description can denote an individual who fails to exist only if among the values of the variables are

things which do not (in the appropriate sense) exist. For example, if among the values of the variable 'x' are all persons who ever lived, and if 'exists' is taken to apply to those persons who are yet alive, then 'the x such that x wrote *Meaning and Necessity*' denotes someone who fails to exist and 'the x such that x wrote *Principia Mathematica*' fails to denote. If the values of the variables are limited to persons now alive, then neither description denotes.

The universe of discourse of a theory need not be limited to the values of the variables. There may well be entities which are not among the values of the variables but which are related to those values in various natural and interesting ways, as books are related to their authors, sets to their members, and ancestors to their surviving descendents. A theory may afford recognition to such entities by mentioning them individually, by name or singular term, without quantifying over them. Much that would otherwise be artificially constrained can thus be treated easily and naturally.

Though our variable binding discourse be limited to natural numbers, we may wish to drop in occasional reference to an unnatural rational, perhaps via the functional expression '$x/2$'. When the values of the variables are so restricted, the following are all true. Why deny them?

$$\exists x \, \forall y \; y \neq x/2$$
$$\forall x \; 2(x/2) = x$$
$$\forall x \, \forall y \, (y = x/2 \leftrightarrow 2y = x)$$

Must '$x/2$' fail to denote when 'x' takes the value 3? Of course not. The reasonable course is to let it then denote 1-1/2. Must 'the y such that $2y = x$' fail to denote when 'x' takes the value 3? Yes.

Homework Problem # 24. In Zermelo-Fraenkel set theory the set of all values of the variables is not among those values. This can be expressed as follows:

$$\sim \exists x \; x = \{y : y = y\}$$

Must '$\{y : y = y\}$' fail to denote? Must 'the x such that $\forall y \, (y \in x \leftrightarrow y = y)$' fail to denote?[23]

Usually it is most convenient to allow the values of the variables to comprehend the entire universe of discourse, marking realms of special interest with predicates. Expressibility increases at no apparent cost. Such motivations lead modal logicians to take as values of their variables all *possible* individuals and to add a predicate of actuality. Similar motivations lead logicians of tense to range their variables over past, present, and future individuals, and to add a predicate of occurrence. But this strategy may entail hidden costs. The systematization of a theory that comes with axiomatization may be lost or compromised. Increased expressibility may open the door to the discussion of issues we shun. In addition, a wider range for the variables may engender talk of new entities in a still wider universe of discourse, with the result that the universe of discourse does not yet close with the domain of values of the variables.

Homework Problem # 25. What happens if the strategy of expanding the domain of values of the variables to meet the universe of discourse is applied to a set theory with abstracts, $\ulcorner\{x:\phi\}\urcorner$, some of which denote sets not among the values of the variables?

We have seen that although our choice of values for the bound variables will restrict the possible values of definite descriptions, there is no sound reason to restrict the values of all terms in the same way. Thus, putting aside the bizarre view of Appendix X, there is nothing to prevent us from treating proper names which denote with respect to some circumstance as denoting the same entity with respect to all possible circumstances, including those in which the entity is not among the values of the variables or, in some other sense, does not exist. The analysis of proper names taken from fiction does not motivate any departure from this practice. I conclude that a proper name either denotes the same individual with respect to every possible circumstance or else denotes nothing with respect to any possible circumstance.

APPENDIX XIII: THE EXCLUSION OF NONDENOTING TERMS

There is an alternative to so-to-speak-denotation which is equally smooth. We can use so-to-speak definite descriptions. An entity, $*$, is chosen from, or added to, the universe of discourse of the language. A slight alteration is made in the definite description operator; now written 'the*'. $\ulcorner$the* $x\ \phi\urcorner$ is translated as 'the unique entity among the values of the variable 'x' which satisfies ϕ; or, if there is none, $*$'.[24] It is clear that 'the*$x(x \neq x)$' denotes $*$. Whatever ease of semantical formulation resulted from the adoption of so-to-speak-denotation also accrues to the adoption of 'the*', provided that a similar alteration is made in the meaning of *all* nondenoting terms.[25]

Let α^* be the altered version of α. It is conceptually important to distinguish the following:

α^* denotes $*$
α so-to-speak-denotes †.

The latter is equivalent to saying that α does not denote; the former holds when α does not denote, but also holds when α denotes $*$. Another aspect of the difference comes out when we ask what considerations are relevant to determining the truth values of atomic sentences. When α does not denote, the considerations relevant to determining the truth value of $\ulcorner\Pi\alpha\urcorner$ (for extensional atomic predicates Π) are very different from those relevant to determining the truth value of $\ulcorner\Pi\alpha^*\urcorner$. The truth value of $\ulcorner\Pi\alpha^*\urcorner$ is fixed by the choice of $*$ and its properties. Determination of the truth value of $\ulcorner\Pi\alpha\urcorner$, and even whether it has one, suffers no such constraints. Since † is alien to the universe of discourse of the object language, its properties are irrelevant. If identity is given its standard interpretation, $\ulcorner\alpha^* = \beta^*\urcorner$ *must* be true when neither α nor β denote, since in that case both α^* and β^* denote the same element of the universe of discourse. But the mere interpretation of identity does not yet determine the truth value of $\ulcorner\alpha = \beta\urcorner$ when neither α nor β denotes. Adoption of so-to-speak-denotation may be a consequence of the decision to call $\ulcorner\alpha = \beta\urcorner$ true, but so-to-speak-denotation also has its uses when $\ulcorner\alpha = \beta\urcorner$ is to be neither true nor false.

It is clear from the interdefinability of 'denotes' and 'so-to-speak-denotes' that the use of the latter for the formulation of the semantical rules does not limit the semantical alternatives for treating nondenoting terms. On the other hand, the use of α^* rather than α, *avoids* the problem of nondenoting terms by confining the object language to terms whose denotation is guaranteed.

Within the systems which exclude nondenoting terms, a variety of altered definite description operators are available. Among those of the form 'the*' some choose $*$ within the values of the variables, some without. An inner choice of $*$ yields a simpler axiomatization of the resulting logic. But it has turned out that the logic resulting from an outer choice of $*$ is much more smoothly axiomatizable than was thought possible twenty years ago. An outer choice of $*$ allows α^* to better simulate α. But the improvement is only to the extent that nondenoting terms are clearly distinguished from terms which denote elements of the domain of values of the variables. The formula $\ulcorner\alpha^* = \text{the}^* x\ x \neq x\urcorner$ does not differentiate non-denoting terms α from those which naturally denote $*$.

There is no general way, within a theory, to absolutely determine whether a term α for which $\ulcorner\sim\exists x\ x = \alpha\urcorner$ is true denotes an element of the universe of discourse or only so-to-speak-denotes †. The distinction is not in general expressible within the language.[26] Even the difference between a choice of $*$ within or without the domain of values of the variables may be disguised by interdefinable alterations of notation which extend or restrict the range of quantification by just that one element. But the intended semantics may often be inferred from theorems of the form $\ulcorner\alpha = \Delta\urcorner$, where Δ is a term which 'naturally' denotes. For example, within a theory of virtual classes, 'the $x(x \neq x) = \{x : x = x\}$' suggests that 'the $x(x \neq x)$' denotes an element of the universe of discourse, whereas such tantalizing assertions as '$\{\text{the } x(x \neq x)\} = \{x : x \neq x\}$' suggest that 'the $x(x \neq x)$' denotes nothing.

The important question is whether we accept the outer entities (those in the universe of discourse but not in the domain of values of the variables) as *real*, as entering into properties and relations of interest to the object language with as much vigor and independence as do the inner entities, lacking only the characteristic property of the inner entities. If we do, then the choice of $*$ as inner or outer seems of secondary importance. If we do not, then there seems no need for more than one outer entity, and its choice as $*$ amounts to identifying it with †.

Homework Problem # 26. Dana Scott has proposed a theory of descriptions according to which the value of an improper description is not an element of the domain of values of the variables.[27] Is he recommending the adoption of so-to-speak-denotation or just an outer choice of $*$?

Homework Problem # 27. 'the $x\,Fx$' denotes the unique inner entity satisfying 'Fx'. If more than one entity satisfies 'Fx', there may still be a unique common value for the functional expression '$g(x)$' whenever the value of 'x' satisfies 'Fx'. Thus in a generalized theory of definite descriptions we may wish an operator of the form 'the $x(g(x) : Fx)$'. So long as the value of '$g(x)$' is an inner entity, this operator is expressed by 'the $y\exists x(Fx \wedge y = g(x))$'. But if the language includes terms such as '$x/2$', which carry inner entities to outer ones, a new operator must be introduced. We write $\ulcorner$the $x_0 \ldots x_n (\alpha : \phi)\urcorner$ for the generalized definite description. The variables $x_0 \ldots x_n$ are bound by the operator. It is permitted that the value of α may be an outer entity. The familiar $\ulcorner$the $x\,\phi\urcorner$ is definable by $\ulcorner$the $x(x : \phi)\urcorner$. A single schema characterizes the generalized definite description:

(L) $\beta \neq \text{the } x(x : x \neq x) \rightarrow$
$[\beta = \text{the } x_0 \ldots x_n (\alpha : \phi) \leftrightarrow \exists x_0 \ldots x_n [\forall y_0 \ldots y_n (\exists x_0 \ldots x_n (\phi \wedge \alpha = \alpha^y_x) \leftrightarrow \alpha = \alpha^y_x) \wedge \alpha = \beta]]$

where α^y_x is the proper substitution of $y_0 \ldots y_n$ for $x_0 \ldots x_n$ in α. Call the schema which results from (L) by restricting attention to the familiar case of the form $\ulcorner$the $x(x : \phi)\urcorner$ '(D)'. Give a simple characterization of the theory of descriptions which results from (D) by adding:

(I). $\exists x(x = \text{the } x(x : x \neq x))$.

Give a simple characterization of the theory which results from adding the negation of (I) to (D). Show that (D) is equipolent to the disjunction of the two theories as you have characterized them. Is any alteration in (L) called for if 'the $x(x : x \neq x)$' is taken as so-to-speak-denoting †?

APPENDIX XIV: A LAST SOLUTION

Take the changing tokens of the more interesting solution and slice them up as in the bizarre view of Appendix X. Now ignore all properties of the slices but their time and type (ignore, for example, their location). We can then reassemble the tokens as in the compromise view of Appendix X. A token can now be thought of as a function which assigns to each moment in its lifetime, its type at that moment. Under this interpretation two tokens with the same type at a given time literally coincide at that time. These tokens are idealized versions of the real tokens (the physical objects afflicted with location and all that) with which we usually deal. To each such real token there corresponds, in the obvious way, an ideal token. Using ideal tokens we can construct a variant of the more interesting solution which is slightly less natural but which may come closer to meeting the adequacy condition: (1) = (2). Treat Index-1 as naming the ideal token which corresponds to Bob-Ted, and similarly for Index-2. The addition of quotation marks becomes an operation directly on the types which constitute the slices of (1) and (2). Otherwise, the argument proceeds as in the more interesting solution. We do not quite achieve the identity of (1) and (2), but almost. At the present time, (1) *coincides* with (2).[28]

Compared to the more interesting solution this solution has the drawback of standing the relation between tokens and types on its head. A consequence of the upside down perspective is that when two real tokens are congruent, their idealizations are coincident. If congruence is as close to identity as coincidence is, then the last solution is no improvement over the more interesting one. From a methodological point of view, however, the last solution is very interesting. Let us look at it as a variant of the complete solution. There, '(2)' was regarded as abbreviating a description which denoted different sentence types at different times. Since applicability of the predicate 'can be changed . . .' depends on the referent of the abbreviated description at times other than the time of utterance (at which time (1) = (2)), it was not surprising that the substitution of '(1)' for '(2)' in this context did not preserve truth. The now common diagnosis of such failures of substitutivity is that substitution in *intensional* contexts like those produced by the 'can be changed...' predicate requires that '(1)' and '(2)' have, not only the same referent, but the same sense.[29] Frege (1952) would agree and go further; within such contexts, '(1)' and '(2)' *refer* to their ordinary sense. When '(1)' and '(2)' are given the interpretation appropriate to their occurrence as subjects of the 'can be changed...' predicate, it will be discovered that the purported identity, (1) = (2), is not a true identity but only a matter of coincidence.[30] Thus we see that the interpretation of Index-1 proposed in the last solution accords exactly with the method of Frege, made explicit by Church, for *completing* the complete solution.

Frege exports intensionality by reinterpreting the expressions which lie within an intensional context. Those which would ordinarily be taken to designate different things with respect to different possible circumstances are reinterpreted to take a fixed designatum, the sense, which by itself determines the entire spectrum of former designata. To put it Kripke's way, a flaccid designator is transformed into a rigid one. But in a way very different from the introduction of a proper name through a dubbing by description. A dubbing by the description α introduces a new expression which rigidly designates the same entity as that which happens to be designated by α with respect to the context of the dubbing. Frege's reinterpretation of α has α itself rigidly designating a new entity of a higher level than any of those which it formerly designated.[31] According to Frege, even an expression in an oblique context is open to substitution by an expression whose entire spectrum is determined by means of the same higher level entity (the same sense). Thus the reinterpretation allows free substitution of expressions whose *reinterpreted* designata are the same. But very few pairs of expressions will pass *that* test.

The process of Fregean ascent can be reversed to import intensionality where none is apparent. Any continuant with different stages in different circumstances, can be sliced into its stages. Any rigid designator of such a continuant can be deinterpreted to designate, with respect to a circumstance, only the then occurrent *stage* of the continuant it formerly designated.[32] The unity of the continuant is dissipated, perhaps irretrievably. It survives primarily in the spectra of the vestigial, no longer rigid, designators. Identity becomes a subject demanding serious attention. Distinct things can be 'the same individual'! Coincidence degenerates to identity. Intensionality runs rampant.

> Although I *am identical with* my body, one of us will survive the other.

Thus begins the long process of Darwinian descent.[33]

University of California, Los Angeles

NOTES

[1] The problem is stated thus in *The Journal of Philosophy* **68** (1971), p. 86, where it is attributed to Professor Richard Cartwright. Solutions follow. Certain collateral issues are discussed in a series of appendices of varying interest. Suggestions for further study are given in the homework problems. An Instructor's Manual is in preparation. All of this has been supported by the National Science Foundation.

[2] I shall use 'refers to', 'denotes', 'designates', 'takes as value', etc., indifferently for the standard notion. Though my way of talking may suggest it, Donnellan's *referential use* is not here applicable.

[3] There is an ellipsis here. The truth of T(Carol) depends on the reference made by T(Carol)'s '(1)'. (Carol may have a remote twin whose '(1)' token is not co-referential with Carol's.) A more explicit form is:

> T(Carol) is not true when T(Bob) is taken as referent of '(1)'.

Or, since T(Carol)'s '(1)' is the only word in T(Carol) whose reference is under examination:

> T(Carol) is not true with respect to T(Bob).

Or since we have fixed *our* use of '(1)' by means of Index-1:

> T(Carol) is not true with respect to (1).

Or, since, as remarked in the preliminaries, it is an assumption of the problem that Carol's use of '(1)' is co-referential with ours:

> T(Carol) is not true.

[4] I waver between Alice and T(Alice) as vehicle of truth. The ambivalence is not critical. The truth value of T(Alice) should, for this problem, be evaluated with respect to the individual referred to by Alice.

[5] Surely on a distinction of such fundamental metaphysical importance, the choice of language in framing the problem was no accident.

[6] The *subjunctive* conditional is not critical to this example or to the following analysis of the problem. We may suppose that M *will* be heated to 200°, and thereby shift to the simple future tense.

When M is heated to 200°, M's length will be more than one meter. The occurrence of 'M's length' remains oblique; it cannot be replaced by its co-designator, 'one meter'. A similar shift from the subjunctive or modal to the future tense would also not affect the following analysis of the problem. It is interesting to note, in comparison, that no intensional context of any form was involved in the preceding solution to the problem. According to that analysis, it is the *present* referent of Index-1, Bob-Ted himself not one of his types or stages, that becomes true.

[7] Anything of which we can frame a definite description can be dubbed by description including, for example, Newman 1 (the first child to be born in the twenty-second century). Thus we might dub by description even when the subject is present, if we are unaware of the fact, or if he is not appropriately 'available', or if we have an ulterior motive.

[8] There was always something implausible about the idea that the referent of a proper name is determined by the currently associated descriptions. For example, the entry under 'Rameses VIII' in the *Concise Biographical Dictionary* (Concise Publications: Walla, Washington) is 'One of a number of ancient pharaohs about whom nothing is known'.

[9] An explicit perpetration occurs in Kaplan (1968, p. 196). But he has not erred alone.

[10] To interpret this theory within a normal one, take the stages to be ordered couples consisting of a moment of time and the coincidence class of one of the normal (continuant) individuals at that time. The coincidence class of a given continuant at a given time is the class of all those continuants which coincide with the given continuant at the given time. The assemblages are determined by the normal individuals. The assemblage corresponding to a normal individual a is that function which assigns to each moment of time at which a occurs the coincidence class of a at that time. Though the value of each occurrence of a variable is a stage, these stages are coordinated by means of assemblages determined by the quantifiers. An existentially quantified formula holds at a given moment if there is an assemblage which has a stage at that moment and which is such that the formula is satisfied by taking as value of each occurrence of the quantified variable the relevant stage of the assemblage. The universal quantifier is, as usual, the dual of the existential. Atomic predicates must also be reinterpreted to apply to the coincidence classes of the continuants to which they originally applied.

[11] See Carnap, (1958, esp. §48) for further discussion of genidentity and its topology.

[12] Since the problem of quantifying out has only recently been solved, the solution to Homework Problem No. 19 is given here, but in a form intended to discourage peeking.

$$\exists t \in \mathbf{T}\, [t < 0 \wedge \forall f \in \mathbf{A}\, [f(t) \in \mathbf{S}(t) \rightarrow (f(t) \in \mathbf{F}(t) \rightarrow f(0) \in \mathbf{F}(0))]]$$

[13] There is a tacit prejudice in this argument. Namely, that the value of a constant with respect to a given moment must be among the values of the variables in variable binding operators evaluated with respect to that moment. I shall attempt to exorcise this prejudice in Appendix XII. Even then, what stage of Aristotle should 'Aristotle' now denote? His birthstage? His deathstage? A triumphant middle-age stage?

[14] The bizarre view is adopted in Kaplan (forthcoming) and Lewis (1968), in neither of which, I fear, is the relation to normal theories correctly seen.

[15] No reason remains other than the prejudice alluded to in note 13, and even given the prejudice, why not let the variables themselves take nonoccurrent assemblages as values? How else to express the fact that I now remember someone who is no longer alive?

[16] Hence, 'the person who both is Quine and is named 'Quine'' would not denote anything with respect to such a world.

[17] Like the token Bob-Ted, the name 'Aristotle' may have been somewhat changed in the course of its travels.

[18] I am not sure that this is how our 'Pegasus' originated but let us assume it so.

[19] Suppose, for the moment, that we take possible individuals, both actualized and unactualized, seriously enough to quantify over them (thus validating ⌜$\Diamond \exists x\phi \rightarrow \exists x \Diamond \phi$⌝). It still does not follow from the fact that if the Ramsified myth had been true there would have been an actualized winged horse, that there is some possible individual such that if the Ramsified myth had been true *he* would have been an actualized winged horse. There are simply too many ways (possible worlds) in which the Ramsified myth might have been true. (The critical invalidity is $[(\phi \Rrightarrow (\psi \vee \chi)) \rightarrow ((\phi \Rrightarrow \psi) \vee (\phi \Rrightarrow \chi))]$ where '$\Rrightarrow$' symbolizes the subjunctive conditional.) Much less does it follow that we could properly speak of *the* possible individual who would have been an actualized winged horse had the Ramsified myth been true. But some such descriptions may be proper. In the most plausible cases we speak of the unique possible individual that would have resulted had a certain closed, developing, deterministic system not been externally aborted. (The possibility of externally induced abortion implies that the system is not completely closed.) Consider, for example, the completely automated automobile assembly line. In full operation, it is, at each moment, pregnant with its next product. Each component: body, frame, motor, etc., lies at the head of its own subassembly line, awaiting only Final Assembly. Can we not speak of the very automobile that would have been produced had the Ecologists Revolution been delayed another 47 seconds?

[20] I will ignore the immediate conjecture that Pegasus symbolizes, and thus 'Pegasus' denotes, *that which man strives for but never fully attains.* Such symbolizations are not reserved to fictional entities; Carnap symbolized the same.

[21] Lewis (1970) would so define theoretical terms of science.

[22] A conversation with my colleague John Bennett caused me to believe this.

[23] Hint: re-read Scott (1967). But see Appendix XIII regarding his answer to the second question.

[24] Note that '$*$' is a symbol of the metalanguage, and 'the*' is an operator of the object language.

[25] In a generalized theory of descriptions (see Homework Problem # 27) this can be accomplished by treating each term α as semantically equivalent to ⌜the x $(\alpha : x = x)$⌝ where 'x' is not free in α.

[26] The problem is that a formal isomorphism can be constructed between a model using † and one in which the universe of discourse is enlarged to include a new element $*$. (Barring, of course, the possible decision to treat ⌜$\alpha = \alpha$⌝ as false, or at least not true, for nondenoting α.) *Given* that the definite description operator of a theory is 'the' not 'the*', the formula ⌜$\alpha =$ the $x(x \neq x)$⌝, which holds only for nondenoting α, can be used. But lacking some notational sign to distinguish the two operators they are in general indiscernable.

[27] Dana Scott (1967). Also see references to other authors therein.

[28] My attention was drawn to this solution by Richard Montague's solution (in 'The Proper Treatment of Quantification in Ordinary English', this volume) to Partee's paradox: from the premises 'the temperature is ninety' and 'the temperature is rising', the conclusion 'ninety is rising' would appear to follow by normal principles of logic; yet there are occasions on which both premises are true, but none on which the conclusion is. Montague has 'the temperature' denote the function which assigns to each moment the temperature at that moment, 'ninety' denote the constant function to ninety, and the putative 'is' of identity (in the first premise) denote the relation of coincidence.

An alternative to Montague's solution, in the style of the complete solution, would take 'the temperature' and 'ninety' both to designate a number (the unit, degrees Fahrenheit, is tacit in the terms); the name rigidly and the description flaccidly. The 'is' of the first premise then *is* the 'is' of identity. The predicate 'is rising' must be regarded as producing an intensional context, but it receives the now standard treatment.

The availability of, and some of the consequences of, certain trade-offs between the reference of terms, the intensionality of contexts, and the like is the subject of this appendix.

[29] Here I take the sense of an expression to be its *intension* in the sense of Carnap (1947), namely that function which assigns to each possible circumstance the denotatum (called, by Carnap, the *extension*) of the expression with respect to that circumstance. Strictly speaking the sense *determines* the intension. The same intension may be determined (in different ways) by different senses.

[30] If f and g are functions, they coincide at a point if their values are the same at that point. If α and β are terms such that ⌜$\alpha = \beta$⌝ is true with respect to a given possible world, then the intension of α and the intension of β will coincide at that world. A predicate expressing coincidence is easily definable in Church's (1951) formalization of Frege's semantics.

[31] To regard an expression other than a proper name as a rigid designator need not entail any unwillingness to recognize the distinctive *syntactical* role played by expressions of differing syntactical categories. Not all rigid designators are, prima facie, proper names; not all are, prima facie, names. Designators like the 'red' in 'Your eye is red' and the 'penguin' in 'Peter is a penguin', which would not ordinarily be regarded as proper names, may yet be rigid if regarded as designating the appropriate entities. If 'red' designates the property of being red, it is probably rigid. If it designates the class of red things, it is certainly not rigid. In my own esoteric doctrines, 'red' rigidly designates a third entity, the color red. Similarly, 'penguin' rigidly designates the species penguin (almost all single words other than particles seem to me to be rigid designators). For Frege, even 'the class of red things' and 'the class of penguins', when located within an oblique context, are rigid designators (though not of classes of red things and penguins).

[32] Just such a process will transform the last solution back into the complete one.

[33] Sam Darwin is the widely acclaimed ontologist and delicatessen operator who once remarked, "Balonies? I don't believe in them. All there is are *slices* arranged in different ways. They come arranged in one way; my job is to rearrange them in tastier ways." The Sam Darwin Fund supports research on the principle of individuation for balonies (what properties of slices determine them as coming from 'the same baloney'). The Fund reports that a breakthrough may be near based on discoveries made with the help of a recently acquired electron microscope. Related investigations, not sponsored by the Darwin Fund, are reported in Geach (1967b), Perry (1970), Lewis (1971), and Perry (forthcoming).

BIBLIOGRAPHY

Ajdukiewicz, K., *Język i Poznanie*, Warsaw 1960.

Austin, J. L., 'Truth' in *Philosophical Papers* (ed. by J. O. Urmson and G. J. Warnock), Oxford 1961.

Austin, J. L., *How To Do Things With Words*, New York 1965.

Bach, E., 'Nouns and Noun Phrases' in *Universals in Linguistic Theory* (ed. by E. Bach and R. Harms), New York 1968.

Bach, E., 'Problominalization', *Linguistic Inquiry* **1** (1970) 121–2.

Bach, E. and Harms, R. (eds.), *Universals in Linguistic Theory*, New York 1968.

Bar-Hillel, Y., 'Logical Syntax and Semantics', *Language* **30** (1954) 230–7.

Bar-Hillel, Y., Gaifman, C., and Shamir, E., 'On Categorial and Phrase-Structure Grammars', *Bulletin of Research on the Council of Israel* **9F** (1960) 1–16.

Becker, E., *An Analysis of Thirty-One Primers*, Unpublished master's thesis, University of Pittsburgh, 1936.

Bell, A., *A State-Process Approach to Syllabicity and Syllable Structure*, Unpublished doctoral dissertation, Stanford University, 1971.

Berman, A., 'Abstract and Agentive Sentences', Unpublished manuscript, Harvard University, 1970.

Bierwisch, M., 'Two Critical Problems in Accent Rules', *Journal of Linguistics* **4** (1968) 173–9.

Binnick, R., Morgan, J., and Greene, G., 'Camelot, 1968', Mimeographed manuscript, 1968.

Bolinger, D., 'Stress and Information' in *Forms of English* (ed. by I. Abe and T. Kanekiyo), Cambridge, Mass., 1958.

Bormuth, J. R., 'Readability: A New Approach', *Reading Research Quarterly* **1** (1966) 79–132.

Braine, M. D. S., 'On Two Types of Models of the Internalization of Grammars' (1971).

Bresnan, J., 'An Argument Against Pronominalization', *Linguistic Inquiry* **1** (1970a) 122–3.

Bresnan, J., 'On Complementizers: Toward a Syntactic Theory of Complement Types', *Foundations of Language* **6** (1970b) 297–321.

Bresnan, J., 'Sentence Stress and Syntactic Transformations', this volume, pp. 3–47.

Bresnan, J., 'The Theory of Complementation in English Syntax', 1971 (in preparation).

Brown, R. and Hanlon, C., 'Derivational Complexity and the Order of Acquisition', in *Cognition and The Development of Language* (ed. by J. R. Hayes), New York 1970.

Burge, T., *Truth and Some Referential Devices*, Unpublished doctoral dissertation, Princeton University, 1971.

Burling, R., 'Language Development of a Garo and English-Speaking Child', *Word* **15** (1959) 45–68.

Carden, G. and Miller, A. G., 'More Problominalizations', *Linguistic Inquiry* **1** (1970) 555–6.

Carnap, R., *Meaning and Necessity*, Chicago 1947. (Enlarged edition 1956).

Carnap, R., 'On Belief Sentences' in *Philosophy and Analysis* (ed. by M. Macdonald), Oxford 1954. Also Appendix C in Carnap (1956).

Hintikka et al. (eds.), Approaches to Natural Language, 519–526.