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NAMES FOR RELATIONS

Peter van Inwagen
The University of Notre Dame

This essay began its life as a sequel to "A Theory of Properties."[1] In that essay, I had said

> A proper presentation of this theory [*sc.* of properties] would treat properties as a special kind of relation. And it would treat propositions as a special kind of relation: it would treat properties as monadic relations and propositions as 0-adic relations. But I will not attempt to discuss relations within the confines of this paper.[2]

The earliest drafts of the intended sequel were entitled "A Theory of Relations." Later drafts were more modestly entitled "Toward a Theory of Relations." My purpose in the earlier drafts was to discharge the promissory note issued in the above quotation: I intended to present a theory of relations that treated propositions as 0-adic relations and properties as monadic relations. My purpose in the later drafts was to present only a part of such a theory, a preliminary sketch of a theory of relations that left many important questions unanswered. The early paragraphs of both sets of drafts contained the following statement: "I will approach the problem of presenting a theory of relations by considering what might seem to be a minor, peripheral problem about relations, the problem of relation-names, or, more exactly, the problem of finding a formal, systematic way of naming relations." It has now become evident to me that the seeming minor, peripheral problem is problem enough for one paper.

It will be evident to at least some readers that the difficulties I encounter in attempting to solve this problem are due to an assumption I refuse to forego, an assumption that has been rejected or severely qualified by the authors of three important papers about relations.[3] This assumption applies to n-adic relations for any $n > 1$. I will state the assumption for the case $n = 2$: Every dyadic relation has at least one converse[4]; there are non-symmetrical dyadic relations; no non-symmetrical dyadic relation is identical with any of its converses. The difficulties I encounter in attempting to find names for relations may be regarded as evidence for the thesis that a workable theory of relations must reject this assumption. I will present some intuitive considerations in favor of the existence of non-symmetrical dyadic relations in Section 5.

1. Names for propositions and properties

I suppose that propositions are 0-adic relations and properties (attributes, qualities, features, characteristics) are monadic (unary) relations. (Polyadic relations—dyadic or binary relations, triadic or ternary relations, and so on—I will call *proper* relations.)[5] If I am right in supposing this, then the "problem of finding a formal, systematic way of naming relations" is a generalization of the "problem" of finding a formal, systematic way of naming propositions and properties. ("Problem" in scare-quotes because the problem of finding a formal, systematic way of naming propositions is so easy to solve that it scarcely deserves to be called a problem. And the problem of finding a formal, systematic way of naming properties is only slightly more difficult and only slightly more interesting.) That is to say, if properties are monadic relations and propositions are 0-adic relations, then the formal, systematic names of properties and propositions ought to fall out of a theory of the formal, systematic names of relations as special cases. Let us therefore begin by looking first at the names of propositions and then at the names of properties.

Propositions, like objects of any kind, can have all sorts of names: 'the Axiom of Choice', for example, or 'Kant's customary example of an arithmetical proposition'. But these are not formal names of propositions. By a formal name of a proposition, I mean a name that *displays* its content, that contains as a part a sentence that expresses that proposition. The usual way of forming a formal name of a proposition is to prefix to a sentence that expresses that proposition the operator 'the proposition that' (which I will abbreviate 'Π').[6] Thus, someone can say, "I don't know what proposition Kant's customary example of an arithmetical proposition is," and make sense. But it makes no sense to say, "I don't know what proposition Π7+5 = 12 is," for to understand the formal name of a proposition is to know what proposition it denotes. It seems obvious that phases of this form denote propositions (given that there are propositions for them to denote), although they may do so only in context: 'Π all men are mortal', when used by a various speakers on a various occasions may denote different propositions, since 'man' can mean various things, such as 'human being' and 'adult male human being'—and 'mortal' can mean either 'does die at some time' or 'could die at some time'. And of course, there are problems about indexicals: it is a matter of dispute whether 'Π it is now raining' always denotes the same proposition or denotes different propositions if it is uttered at different times. Still (always assuming that there are propositions), the contention that prefixing 'the proposition that' to a sentence that expresses a proposition denotes a proposition seems no more problematical than, say, the contention that the result of prefixing 'the father of' to a term that denotes a human being who has a father denotes a father, and that's not a terribly problematical statement. I note that (even if there are propositions) not all phrases formed by prefixing 'the proposition that' to a sentence that purports to express a proposition denote a proposition, since a sentence that purports to express a proposition may not. It would seem, for example, that (*pace* Josiah Royce) 'the proposition that the world is a progressively realized community of interpretation' does not denote anything. And it would seem, and more than seem, that the expression,

> The proposition that the proposition named by the only proposition-name whose initial words are 'the proposition that the proposition' that occurs in van Inwagen's "Names for Relations" is false

does not denote anything.

There is no formal reason to restrict the application of the operator 'Π' to closed sentences. 'Π *x* is wise' is an open term (like 'the father of *x*'). It may thus be "quantified into": 'For some *x*, Π *x* is wise is true'. Any reason for rejecting such constructions would have to be other than formal—presumably an adherence to a philosophical theory (anti-haecceitism, perhaps).

Such formal names for propositions are *systematic* in this sense: in forming names by this method we use only one device, and it always "works": the sentence-operator 'Π' is that only piece of vocabulary we need, and any proposition anyone can refer to by any means can be referred to by this means. (Which is not of course to say that just anyone who is able to refer to a certain proposition as 'Kant's customary example of an arithmetical proposition' can also refer to it by using a phrase of the form 'Π ... '.)

Let us now turn to the names of properties. In addition to having names like 'the property that, according to Frege's famous account of it, is a property of not of objects but of concepts', properties have formal names like 'existence', 'wisdom', and 'the property of being white'. These are formal names in the sense that they reveal or make manifest the property they denote. Or, more cautiously, they are names that purport to denote properties and purport to reveal or make manifest the properties they denote. One can sensibly say, "I wasn't following the lecture because I didn't know what property the property that, according to Frege's famous account of it, is a property of not of objects but of concepts was; eventually I realized that it was existence." One can't sensibly say anything that starts, "I wasn't following the sermon because I didn't know what property wisdom was; eventually I realized that it was . . . ." Or, rather, one *can* sensibly say things that start that way, but, if one does and if what one says is indeed sensible, one's statement has to be understood as containing invisible or unpronounced quotation marks (" . . . eventually I realized that it was what we call in French *sagesse*"; and, of course, one can say things like "I know that there's supposed to be such a thing as wisdom, but I've never been able to make out what people mean by the word."). But such formal names of properties, the formal names of properties that crop up in ordinary discourse, do not follow any systematic pattern.

Can we devise an operator that does with respect to properties what 'the proposition that' does with respect to propositions? Since properties are expressed by sentences in which one variable is free, such an operator would (one supposes) have to be a variable-binding operator. As a quantifier takes a sentence with one free variable and makes a closed sentence, so the operator we are looking for would take a sentence with one free variable and make a closed term.[7]

One operator that will not do for this purpose (I mention it because I shall presently discuss an analogous operator in the case of systematic names for proper relations) is this: 'the property that . . . has if and only if'. This will not do because, for example, the phrase 'the property that *x* has if and only if *x* is white' does not (necessarily) denote whiteness because it does not necessarily denote anything. If there is some property other than whiteness that happens to belong to all white things and only to white things, the phrase will denote nothing. And, even if there happens to be no other property co-extensive with whiteness, it is obviously wrong to adopt a theory of properties that implies that there *cannot* be two (nameable) properties that are co-extensive. Faced with this problem, one might try a "modal fix": one might consider the operator 'the property *y* such that, necessarily, *x* has *y* if and only if'. But this solution to the problem fails if some open sentence expresses a property that is necessarily coextensive with some distinct property. It would, I contend, be wrong to build the very strong (and very controversial) thesis that necessarily coextensive properties are always identical into a proposal about how to name properties systematically. If we had some decisive logical or metaphysical reason to think that necessarily co-extensive properties must be identical, this consideration would, of course, lose its force. But I do not know how to individuate properties, and I will not build a principle of individuation for them into my proposal about how to form systematic names for them. (We should have faced similar problems in the case of propositions if, instead of 'the proposition that' we had considered 'the proposition that is true if and only if' or 'the proposition *x* such that, necessarily, *x* is true if and only if'.)

Here is a better suggestion: a systematic way of constructing formal names of properties should employ the operator 'the property of being an *x* [or what variable you will] such that' (abbreviated 'Π*x*'). Thus 'Π*z* *z* is wise' and 'Π*x* *x* loves *x*'s honor more than *x* loves *x*'s life' and 'Π*y* *y* is either the daughter of a nineteenth-century president of the United States or the husband of a famous marine biologist who is the mother of some of but not all *y*'s children' are property-names. The first denotes the property we ordinarily call 'wisdom'; the second denotes a property that could also be called 'loving honor more than life'; the third denotes a property that has no very manageable name in any natural language. (In saying these things, I assume that there *are* properties and that properties "abound." I assume, that is, that the result of writing 'Π', a variable, and a sentence in which that variable and that variable alone is free denotes a property "in most cases." For a discussion of the qualification "in most cases," see "A Theory of Properties," p. 115. I shall advance a similar thesis about relations in Part 3, albeit in connection with an unsatisfactory account of relation-names.)

There is no formal reason to rule out prefixing 'Π*x*' to a sentence in which variables other than '*x*' are free—no formal reason to rule out forming open terms like 'Π*x z* is wiser than *x*' and sentences like 'For some *z*, Crito has Π*x z* is wiser than *x*'. (Compare our earlier remark about 'Π *x* is wise'.)[8]

2. Names for proper relations

If we are able thus to provide formal, systematic names for propositions and properties (0-adic and monadic relations), how shall we provide formal, systematic names for relations—for *proper* relations?

Let us begin with the case of dyadic relations. It would seem that if we are to provide a systematic way of constructing formal names for dyadic relations, we need to find a sentence-operator that specifies two variables: an operator that, when it is applied to a sentence in which just exactly the two variables it specifies are free, yields a closed term, a term that denotes (or purports to denote) a dyadic relation—the very relation that is "expressed" by the sentence to which it has been applied.[9] Suppose that 'Π' is this operator. (If 'Π' followed by no variables is used to form names of propositions and 'Π' followed by one occurrence of a variable is used to form names of properties, it seems fitting to use 'Π' followed by occurrences of two variables to form names of dyadic relations.) Then 'Π*yz z* loves *y* more than *y* loves *y* or *y* loves *z*' names, or purports to name, a dyadic relation.[10] The operator 'Π' will, moreover, have to bind the variables of the sentence to which it is applied "in the order it specifies." That is, the closed-term-maker 'Π . . . . . . ' is in this respect like the closed-sentence-maker '∀ . . . ∃ . . . ' (and not like '∀ . . . ∀ . . . '): just as '∀*y* ∃*z*' is not equivalent to '∀*z* ∃*y*', 'Π*yz*' is not equivalent to 'Π*zy*'. So, for example, the relation Π*yz y* loves *z* is not identical with the relation Π*zy y* loves *z*.[11] (Or, at any rate, nothing we have said implies that they are. If they are identical, their identity would have to be a consequence of something having to do with the metaphysics of relations—and perhaps something having to do with love.) And we shall want the fact that 'Π' binds variables "in the order it specifies" to be true in a particular way: we shall want the relations named by these two expressions will be mutually converse: if the first names the relation we call "loving" (a big 'if'; I'm simply casting about for an informal relation name for the sake of the example), the second will express its most salient converse, the relation we call "being loved by."

Do we know of any operator that has the features we want 'Π' to have? Well, yes and no.

Consider the binary abstraction operator of higher-order logic, 'λ^2'. (The superscript, hereafter omitted, indicates that the operator is to be immediately followed by occurrences of two variables.). This operator might seem to have just the properties we require of 'Π'. That was Yes. Here's No: It's not at all clear what this operator means. What do names like

λyz *z* loves *y*

and sentences like

(λyz *z* loves *y*) Sally, Harry

mean? (I ignore, for the present, the fact that, strictly speaking, the expression 'λyz *z* loves *y*' has, as the offset sentence shows, the grammar of a predicate and not a name.) Well, the offset sentence means *something* like this: Sally and Harry stand in the relation "loving"; and they stand in that relation in just this way: the latter loves the former—and not necessarily in this way: the former loves the latter. But this statement is at best *suggestive* of a meaning. Can we find some way of reading the binary (ternary . . .) abstraction operator in English—or at least in what Dorothy Grover has called philosopher's English—, a reading whose meaning is as clear as the meanings of 'the proposition that' and 'the property of being a(n) . . . such that' (these are, in effect, our readings of 'λ^0' and 'λ^1' in philosopher's English).

I am inclined to think that we can't. At any rate, I know of no way to turn any of the *n*-ary abstraction operators (other than 'λ^0' and 'λ^1') into English or even philosopher's English. One idea that can be immediately discarded is this: that the operator

The relation that holds between . . . and . . . if and only if

will serve as an English reading of 'λ . . . . . . '. This idea—the Bad Idea, I'll call it—must be discarded for two reasons. I'll call them the Good Reason and the Even Better Reason. Here is the Good Reason. 'The relation that holds between *x* and *y* if and only if *x* has the same number of sides as *y*' *may* name a relation—but it does not if there are two or more numerically distinct relations that hold between *x* and *y* if and only if *x* has the same number of sides as *y*. (Perhaps "having the same number of sides" and "having the same number of interior angles" are two such relations.) We have already mentioned a special case of the Good Reason (as it applies to monadic relations or properties). A "modal fix" in the case of proper relations will be objectionable for a reason that is essentially the same as the reason we mentioned in connection with properties.

The Good Reason is indeed a good reason for discarding the Bad Idea, but it wouldn't be if coextensive, or, more plausibly, necessarily coextensive, relations were always identical. The Even Better Reason would be a decisive reason for discarding the Bad Idea even if necessarily coextensive relations were identical. (Given that there are non-symmetrical relations: the Even Better Reason presupposes that there are relations that have distinct converses. The Good Reason applies both to symmetrical and non-symmetrical relations.)

And the Even Better Reason is this. 'The relation that holds between *y* and *z* if and only if' does indeed bind the variables '*y*' and '*z*' in sentences to which it is applied—but *not* "in the order specified." It is thus comparable to '$\forall y\ \forall z$' and not to '$\forall y\ \exists z$'. This operator, therefore, cannot distinguish between a dyadic relation and its converses. If there are dyadic relations at all—here I make the controversial assumption—, there are non-symmetrical dyadic relations, and a non-symmetrical dyadic relation is not identical with any of its converses. Suppose that 'the relation that holds between *y* and *z* if and only if *y* is to the north of *z*' designates a particular relation, R_1. Let R_2 be a converse of R_1. If either R_1 or R_2 "holds between" two objects (Denmark and Italy, say) so does the other. (If we think informally of R_1 as "being to the north of" and R_2 as "being to the south of," it is evident that these two relations "hold between" the same pairs of objects.) And if that is so, 'the relation that holds between *y* and *z* if and only if *y* is to the north of *z*' is an improper description and does not designate the relation R_1 after all. The problem, of course, is created by the fact that a predicate like 'holds between Denmark and Italy' does not specify the order in which those two nations are to enter into the relation of which it is predicated: 'holds between Denmark and Italy' and 'holds between Italy and Denmark' are synonymous. The logical predicate 'holds between Denmark and Italy' is in this respect like the spatial predicate 'stands between Denmark and Italy': Anything (a mountain, say) that stands between Denmark and Italy must also stand between Italy and Denmark. This problem is not peculiar to the expression 'holds between . . . and . . . '; it will arise for any expression that uses 'and' to connect terms. When 'and' is used to connect singular terms, it makes a plural term (a plural-referring expression) out of them ('Russell and Whitehead' out of 'Russell' and 'Whitehead'.). And the order in which the singular terms occur within such a plural term is of no semantical significance. For this reason, such phrases as 'the relation that holds between *y* and *z* if and only if *y* is to the north of *z*' and 'the relation in which *y* and *z* stand just in the case that *y* is to the north of *z*' are essentially unsuited to be English readings of '$\lambda yz\ y$ is to the north of *z*'.

Might we surmount this difficulty—the indifference of 'and' to displayed order—by brute force? Might we not simply *specify* that the order in which terms are displayed in a "holds between" sentence is the order in which (according to that sentence) they enter into the relation whose name is the subject of the sentence? Might we not simply include in the sentence some explicit phrase like 'in that order'? For example:

The relation *z* holds between Denmark and Italy in that order.

And might we not insist that this sentence is not equivalent to

The relation *z* holds between Italy and Denmark in that order?

If this is correct (and if for some reason we need not worry about the possibility of distinct, co-extensive relations like "having same number of sides" and "having the same number of interior angles"), we can refer to a certain relation as 'the relation that holds between *x* and *y* in that order if and only if *x* is to the north of *y*'; and this phrase and the phrase 'the relation that holds between *y* and *x* in that order if and only if *x* is to the north of *y*' would denote distinct and mutually converse relations. It might be objected that the phrase 'in that order' creates a non-extensional context, analogous to the context created by '. . . was so-called because of his size'. (It seems reasonable to say that the plural terms 'Italy and Denmark' and 'Denmark and Italy' are semantically equivalent; substituting one of these terms for the other in expressions in which they are followed by 'in that order' will sometimes change the semantical value of that expression.) But this objection, well taken or not, could be evaded by specifying order in a way that does not refer to the order in which terms occur in the sentence that contains the specification. One might try something like this: 'the relation that holds between *x* and *y* in the order "*y* first, *x* second" if and only if *x* is to the north of *y*'. But what *is* it for a relation to hold between—for example—Italy and Denmark in the order "Denmark first, Italy second"? You may well ask.

Is it possible to find some English reading for phrases containing the abstraction operator, phrases like 'λyz *y* is to the north of *z*', that do not involve the order-indifferent term-maker 'and'? We might think of such phrases as 'the relation such that *y* bears it to *z* if and only if'.[12] It is perhaps not evident that 'the relation such that *y* bears it to *z* if and only if' is equivalent to 'the relation such that *z* bears it to *y* if and only if'. Even if this suggestion "works" for dyadic relations[13], however, it cannot be extended to triadic relations—or, more generally, to *n*-adic relations, $n > 2$. For what predicate—what predicate corresponding to '. . . bears . . . to . . . ', which is supposed to express the relation in which a dyadic relation and its two relata stand—should we use to express the relation in which a triadic relation and its three relata stand? There would seem to be only two possibilities:

. . . and . . . bear . . . to . . .
. . . bears . . . to . . . and . . . .

Both expressions contain the problematical 'and':

Denmark and Switzerland bear . . . to Italy

Denmark bears . . . to Switzerland and Italy

are equivalent to, respectively,

Switzerland and Denmark bear . . . to Italy

Denmark bears . . . to Italy and Switzerland.

When all is said and done, there seem to be no analogues of 'the proposition that' and 'the property of being a(n) . . . such that' that can be used to form formal and systematic names for proper relations. That is to say, there seems to be no way to find an acceptable English reading of the *n*-ary abstraction operator for the cases $n > 1$.

The problem of finding in English (or any natural language) a formal, systematic device for naming proper relations has no obvious solution.

3. The Problem with Abstraction

All right—the interlocutor concedes—we can't translate abstraction-operator names like 'λyz *y* loves *z*' into philosopher's English. But—the interlocutor protests—so what? We understand them, don't we?

In this section I will try to show that we do not. I will try to explain what kind of understanding we have of sentences like

(λyz *z* loves *y*) Sally, Harry

and try to explain why that kind of understanding isn't enough. The offset sentence purports to name a relation and it tells us that Harry and Sally enter into that relation in a certain order. That is, it is to be distinguished from the sentence

(λyz *z* loves *y*) Harry, Sally—

for it, the former sentence, is true just in the case that Harry loves Sally, and the latter sentence is true just in the case that Sally loves Harry. So, assuming that we know whether Harry loves Sally and whether Sally loves Harry, we know the truth-values of each of these sentences. But that doesn't mean that we understand them. I shall find it easier to defend this thesis if I provide a slightly different formal apparatus—a variant on λ-abstraction—for referring to relations and for making statements about how things enter into them.

To begin with, I want my names for relations to be unequivocally *names*, to occupy nominal—not predicate—positions. Thus I shall want to replace the sentences above with something like

Sally and Harry enter into λyz *z* loves *y*

and

Harry and Sally enter into λyz *z* loves *y*.

The word 'and' in these sentences is not the order-indifferent English plural-term maker that was discussed in the previous section. If it were, the two sentences would be equivalent, and I do not mean them to be. In these sentences, 'and' has the same function as the comma in '(λyz *z* loves *y*) Sally, Harry'; think of it as a comma that is pronounced 'and'. A more important point: terms that follow 'enter into' in such sentences are true denoting terms. They have the grammar of nouns, not predicates. That is, they are subject to (nominal) existential generalization. Thus, '$\exists x$ Harry and Sally enter into x' follows from 'Harry and Sally enter into λyz z loves y' by existential generalization.

I will make one other change in the formal apparatus of λ-abstraction, more for aesthetic reasons than any other. Consider the expression 'λyz z loves y'. If this expression indeed names a relation, the relation it names has many other names: 'λzy y loves z', 'λxy y loves x', 'λzx x loves z', ... These names are "alphabetic variants" on one another—like the sentences '$\exists z$ $\forall x$ Fzzx' and '$\exists x$ $\forall y$ Fxxy'. The formal apparatus I will introduce will avoid this meaningless proliferation of variant names.[14]

I begin with some definitions.

Let us say that a *predicate* of a given natural language is any declarative sentence of that language or any expression formed from a declarative sentence of that language as follows: some of or all the occurrences of terms in that sentence are to be replaced by occurrences of the first n numerals, starting with '**1**'. (We write the numerals in bold-face. '**1**' is the *first numeral*, '**2**' the *second numeral*, and so on.) Thus, a predicate may contain no bold-face numerals; or it may contain '**1**' and no other numerals; or it may contain '**1**' and '**2**' and no other numerals; or it may contain '**1**' and '**2**' and '**3**' and no other numerals—and so on. For example, since 'Miami is north of Boston' is a sentence of English, the following expressions are predicates of English:

Miami is north of Boston

1 is north of Boston

Miami is north of **1**

1 is north of **1**

1 is north of **2**

2 is north of **1**.

A predicate that contains the first n numerals and no others is an *n-place* predicate. (A declarative sentence, an expression containing no bold-face numerals, is a 0-place predicate.) Thus, the second, third, and fourth of the six predicates I have just displayed are one-place predicates, the last two are two-place predicates, and '**2** has married **1** more times than **3** has married **2**' is a three-place predicate.

We now introduce *canonical relation-names*. (By a 'relation-name' I mean a term that *purports* to denote a relation. Thus, 'Kripke's favorite relation' is a relation-name even if Kripke does not in fact have a favorite relation.) A canonical relation-name (of English) consists of the operator 'R' prefixed to a predicate (of English). (Round brackets will be used to indicate the scope of 'R'—the predicate to which it applies.) The operator R thus forms terms from predicates; one might call it an abstraction operator, but only in a loose sense, since it is not a variable-binding operator. In other words, or other symbols, for any predicate P, **“**R(P)**”** is a canonical relation-name (and only expressions so formed are canonical relation-names). (I use large, bold-face double quotes as quasi-quotation marks; they are the same as “Quine corners.”[15])

For any predicate P, **“**R(P)**”** is a canonical

—*proposition-name* if P is a 0-place predicate (a declarative sentence)

—*property-name* if P is a 1-place predicate

—*proper-relation-name* if P is a 2-or-more place predicate

—*dyadic-relation-name* if P is a 2-place predicate[16]

—*triadic-relation-name* if P is a 3-place predicate

. . . and so on. For example, ‘R(Miami is north of Boston)’, ‘R(Miami is north of **1**)’, ‘R(**2** is north of **1**)’, and ‘R(**2** has married **1** more times than **3** has married **2**)’ are canonical relation-names—respectively, a 0-adic-, a monadic-, a dyadic-, and a triadic-relation-name. Where a variable-binding abstraction operator presents us with a plethora of equivalent names—‘λyz z loves y’, ‘λzy y loves z’, ‘λxy y loves x’, ‘λzx x loves z’, . . . , —“predicate abstraction” gives us only one. (In this case, ‘R(**2** loves **1**)’.)

What I have said raises three important questions (at least): Which relation-names have denotations?; What is the nature of these objects, these relations, that relation names denote?; How are these objects, these relations, to be individuated? I shall have a little to say about which relation-names denote and nothing to say about the nature or individuation of relations.

Sentences of the forms displayed by the following list of expressions will be called *relation sentences*. (In these expressions, subscripted ‘t’s represent places at which terms—either variables or phrases that purport to denote particular objects—can occur. Subscripted ‘r’s represent places at which canonical relation-names can occur. An ‘r’ with subscript n represents an n-adic canonical relation-name)

Enter into r_0

t_1 enter into r_1

t_1 **and** t_2 enter into r_2

t_1 **and** t_2 **and** t_3 enter into r_3

.
.
.

(The word '**and**' is bold-faced in order to avoid ambiguities that might arise if it were unclear whether an occurrence of 'and' was a part of a term.[17] We omit the bold-facing in cases in which no such ambiguity threatens.) Relation sentences of the first form are *0-place* relation sentences, of the second form, *1-place* relation sentences, of the third form, *2-place* relation sentences, and so on. It will be noted that an n-place relation sentence actually contains $n + 1$ term-places: the place occupied by the final or "relational" term of a relation sentence does not enter into its "place count." We call the terms of a relation sentence other than its relational term its *proper* terms. An n-place relation sentence thus contains n occurrences of proper terms. (Of course, any of or all the proper terms of a relation sentence may be a relation-name, since relations enter into relations; but the relational term must be a canonical relation-name.)

I introduce some informal stylistic conventions for writing 0- and 1-place relation sentences.

For any declarative sentence (0-place predicate) P, instead of **"**Enter into R(P)**"** one may write **"**The proposition that P is true**"**.

For any term t and one-place predicate P, instead of **"**t enter into R(P)**"** one may write, for any variable α, **"**t has the property of being an α such that Pα**"**, where Pα is the result of replacing all occurrences of '**1**' in P with α.

Thus,

> Socrates enter into R(**1** is snub-nosed),

may be replaced by

> Socrates has the property of being an x [or what variable you will] such that x is snub-nosed

(which may then be abbreviated as 'Socrates has Πx x is snub-nosed').

These conventions may be applied (mutatis mutandis) to canonical 0-adic- and monadic-relation-names when they do not appear in the context of a relation sentence. For example, 'R(**1** is snub-nosed)' is conventionally written 'Πx x is snub-nosed' (or what variable you will).

The proper terms of a relation sentence are designated individually as follows. For any 1-or-more-place relation sentence

> t_1 and t_2 and ... and t_n enter into r_n,

t_1 is the sentence's *first-place term*, t_2 its *second-place term*, ... and t_n its *n-th place term*. Thus the first-place term of

> John and x and John enter into R(**1** loves **3** and **2**)

is 'John', its second-place term is 'x' and its third-place term is 'John': 'John' is thus both its first-place term and its third-place term. (And, of course, the sentence has no fourth-or-higher-place term.)

A relation sentence is intended to have the following property (speaking very loosely): the order of its proper terms displays the order in which the objects they designate enter into the relation designated by its relational term. But what does this mean? I know of only one way to respond to this question. I will present a semantics for relation sentences. (It is not essentially different from the standard semantics for a sentences consisting of a property-name formed by use of the variable-binding abstraction-operator followed by a string of terms.) This semantics will answer the question "But what does this mean?", insofar as the question has an answer, by showing how to calculate the truth-values of, e.g., 'Sally and Harry enter into R(**1** loves **2**)' and 'Harry and Sally enter into R(**1** loves **2**)'; and it will have the consequence that this method will yield different truth-values for the two sentences if either Harry or Sally loves the other and his or her love is not returned.

The semantics turns on the notion of the "non-relational counterpart" of a relation sentence—roughly speaking, the notion of a sentence that is semantically equivalent to that relation sentence and contains its proper terms and no other terms.

For any *n*-place predicate P, the *non-relational counterpart* of a relation sentence whose relational term is **"**R(P)**"** is the result of replacing every occurrence of '**1**' in P with the first-place term of the relation sentence, every occurrence of '**2**' in P with the second-place term of the sentence, . . . , and every occurrence of the *n*th numeral in P with the *n*th-place term of the sentence. For example, the non-relational counterpart of

Enter into R(Miami is north of Boston)

(that is, of 'The proposition that Miami is north of Boston is true') is

Miami is north of Boston.

The non-relational counterpart of

z enter into R(**1** loves **1**)

(that is, of '*z* has Π*x* *x* loves *x*') is

z loves *z*.

The non-relational counterpart of

John and *x* and John enter into R(**1** loves **3** more than **3** loves **2**)

is

John loves John more than John loves *x*.

And here is the semantics:

> If the relational term of a relation sentence denotes nothing, that sentence is false if closed and unsatisfied if open.
>
> Otherwise—if its relational term has a denotation—a relation sentence is semantically equivalent to its non-relational counterpart. That is to say, if it is closed, it has the same truth-value as its non-relational counterpart; if it is open, it is satisfied by the same (sequences of) objects.[18]

The first clause of this semantics is important, for it allows us to escape Russell's Paradox. Relation sentences, like any sentences, can be used to form predicates. Therefore '**1** enter into **1**' and its negation are predicates. And, therefore, 'R (∼**1** enter into **1**)' is a relation-name.[19] Consider the relation sentence

> R (∼**1** enter into **1**) enter into R (∼**1** enter into **1**).

If our semantics for relation sentences consisted solely of the second clause above (minus the initial "otherwise" qualification), we should be in trouble, for the non-relational counterpart[20] of this sentence is its negation! We avoid paradox as follows: we assume—the assumption is entirely *ad hoc*—that there is no such relation as R (∼**1** enter into **1**) (that is, that 'R (∼**1** enter into **1**)' denotes nothing). That allows us to say (by the first clause) that our queer sentence is simply false (and its negation simply true)—and to say that the second clause does not apply to it.

This *ad hoc* maneuver, of course, raises a pointed question: When do canonical relation-names have denotations and when do they not? What can I say? I'd like to suppose that the Platonic heaven is abundantly, rather than sparsely, populated with relations. I'd therefore like to say that *all* meaningful canonical-relation names denote (and denote relations). I can't say that however—not if I say, and I do say, that 'R (∼**1** enter into **1**)' does not denote anything. I am sorry to have to tell you that my only reply to the pointed question is: canonical relation-names have denotations only if their having denotations does not lead to Russellian paradox. (But not, of course, *if*. No proposition is denoted by 'Π the world is a progressively realized community of interpretation'; No property is denoted by 'Πx x is a unicorn'; no dyadic relation is denoted by 'R (Atlantis is north of **2** and south of **1**)'.) I wish I could say something more useful, but, really, no one knows what to do about Russell's paradox. I can say only this. If a sentence expresses a proposition ('Atlantis is north of Bermuda and south of Iceland' does not express a proposition), then, for any predicate P formed from that sentence, **"**R(P)**"** denotes a relation "in most cases."

I am tempted to say that this semantics represents a relation sentence as having the following feature: the order in which its proper terms occur displays the order in which the objects they designate enter into the relation designated by its relational term. I am tempted to say this but I will not, because it's not clear what it means. I will instead make this more modest statement: the semantics treats the order in which the proper terms of a relation sentence occur as semantically significant. The two sentences

Tom and Tim enter into R(**1** is taller than **2**)

Tim and Tom enter into R(**1** is taller than **2**),

are—given that R(**1** is taller than **2**) exists—semantically equivalent to, respectively,

Tom is taller than Tim

and

Tim is taller than Tom.

We could have chosen a technical vocabulary that highlighted the semantical significance of the order in which the proper terms of a relation sentence occur. Instead of the form of words 't_1 and . . . t_n enter into r_n' we could have used, say, 't_1 and . . . t_n enter into r_n in that order'. But this, of course, is no more than a matter of terminology.

We now have a semantics for relation sentences. That, of course, is not much. Even as a semantics it is pretty limited. It does not tell us the satisfaction conditions for sentences that are like relation sentences save that their final term is a variable or an "open" canonical relation name (like 'R(**2** is between *z* and **1**)'). It does not apply to sentences that contain canonical relation names but are not relation sentences—for example, 'R(**1** loves **2**) is a dyadic relation', 'R(**1** loves **2**) and R(**2** loves **1**) are converse' and 'R(**1** is an uncle of **2**) is a Peircean product of R(**1** is a brother of **2**) and R(**2** is a parent of **1**)'. It could, however, be extended to such sentences, using devices borrowed from the model theory of higher-order logic with abstraction. The question is, does this semantics enable us to understand canonical relation names?—or would an extension of it that treated a larger class of sentences containing canonical relation names enable us to understand them?

In my view, the answer to both questions is No. Let us consider a very simple case, the two names 'R(**1** loves **2**)' and 'R(**2** loves **1**)'. If there are indeed relations, and if these names denote relations, they denote two distinct relations—since they denote mutually converse non-symmetrical relations: *x* and *y* enter into R(**1** loves **2**) if and only if *y* and *x* enter into R(**2** loves **1**). But do we really *grasp* these two relations? Have we any idea what objects it is that the two names denote? We perhaps believe that there is a non-symmetrical relation called (informally) "loving." But if the relation is non-symmetrical, it has a distinct converse (or more than one). What do we call this converse (or the most salient of them, if there is more than one)? Perhaps we call it "being loved by." But which of R(**1** loves **2**) and R(**2** loves **1**) is "loving" and which is "being loved by"? Is R(**1** is to the north of **2**) "being to the north of" or "being to the south of"? And these are questions about mere dyadic relations. There are only two permutation operations on dyadic relations, "identity" and "converse." This is a special case of the general truth that there are $n!$ permutation operations on an n-adic relation. There are thus twenty-four ways (including identity) of permuting a quadratic relation like R(**3** loves **1** more than **4** loves **2**). Can you distinguish this relation from its permutation R(**3** loves **2** more than **1** loves **4**)? Our semantics tells us that Tom and Harry and Mary and Tom enter into the former if and only if Mary loves Tom more than Tom loves Harry, and it tells us that they enter "in that order" into the latter if and only if Mary loves Harry more than Tom loves Tom. That is, we know how, using the semantics, to calculate the truth-values of relation sentences with these two relational terms. But—it seems to me—we have no idea what these sentences mean or what the relational terms refer to. Our grasp of the name 'R(**3** loves **1** more than **4** loves **2**)'—even our grasp of 'R(**2** loves **1**)'—thus stands in stark contrast to our grasp of the names 'R(Tom loves Mary)' ('The proposition that Tom loves Mary') and 'R(**1** loves Mary)' ('The property of being an x such that x loves Mary'). We understand "abstraction names" of propositions and properties. We do not understand abstraction names of proper relations. At any rate, we do not understand abstraction names of dyadic relations unless those relations are symmetrical; and we do not understand abstraction names of triadic (etc.) relations unless those relations are indifferent to permutation.[21]

4. Evading the problem: eliminating relations in favor of properties of sequences

Proper relations (hereafter, in this section, relations) are generally said to be exemplified or instantiated or satisfied by sequences of things. For each n-adic relation, $n > 1$, R^n, therefore, there will be the property of satisfying R^n, Πx x is an n-term sequence and x satisfies R^n. (At any rate, there will be this property if—as I assume—properties abound.) Call this property the relation's *property counterpart*. Now (obviously) a sequence *satisfies* R^n if and only if it *has* the property counterpart of R^n. Why not take advantage of this obvious fact and collapse the two things into one? More exactly, why not eliminate relations in favor of their property counterparts? Why not say that (i) there are, strictly speaking, no relations, and (ii), don't worry, because their property counterparts are available to do all the work that philosophers had mistakenly supposed they did? It is of course true—if you want to be picky about the matter—that if there is no such thing as the relation R^n, there is no such thing as the property counterpart of the relation R^n. If there is no such thing as the relation "to the north of", there is no such thing as Πx x is a two-term sequence and x satisfies the relation "being to the north of." There is, nevertheless, *this* property:

> Πx x is a two-term sequence [that is, an ordered pair] and the first term of x is to the north of the second term of x.

The proposal under consideration is that we eliminate relations from our ontology in favor of such properties as these. Let us call these properties *relational stand-ins*—or simply stand-ins. The offset expression can serve as a model for formal, systematic names of stand-ins. The general form of such names (*canonical* names of stand-ins) is:

> Πx [or what variable you will] x is a(n) n-term sequence and S,

where, of course, the number n ($n > 1$) is specified and S is a sentence that contains 'the first term of x' and 'the second term of x' and no other such open names if $n = 2$, 'the first term of x' and 'the second term of x' and 'the third term of x' and no other such open names if $n = 3$, and so on.

The technical terminology that is used to describe relations ('symmetrical', 'dyadic', 'converse' and so on) is easily adapted to the task of describing stand-ins. (The definitions are obvious.) Note that if **R** is a non-symmetrical dyadic stand-in and if R is a canonical stand-in name that denotes either **R** or a converse of **R**, there can never be any confusion about *which* of the two, **R** or the converse, R denotes. One canonical name of a converse of the stand-in named by the offset name above is,

> Πx x is a two-term sequence and the second term of x is to the north of the first term of x,[22]

and it is evident that each of these names denotes a converse of the stand-in the other denotes.[23]

The proposal we are considering—that relations be eliminated in favor of stand-ins—solves the two problems that we failed to solve when we tried to find formal, systematic names for relations. That is to say, no analogue of either problem confronts the present proposal. We have in philosopher's English formal, systematic names for properties, and hence for stand-ins. And, as we have seen, it will in every case be clear that the canonical name of a non-symmetrical dyadic stand-in does not denote any converse of that stand-in.

What of propositions and properties, which, as I have said, I should like to identify with, respectively, 0-adic and monadic relations? This idea—or the counterpart of this idea, for we are now supposing that there are no relations—will have to be abandoned. Let us first consider properties. Stand-ins *are* properties. In what sense then, could we say that a property was to be identified with a stand-in? Can we say that a property is identical with its "monadic stand-in"? For example, might the property Πx x is red be identical with Πx x is a one-term sequence and the (first and only) term of x is red? Not, certainly, on the standard set-theoretical understanding of 'sequence'; on that understanding, the one-term sequence whose member is x is the set whose sole member is x, and the property Πx x is red is obviously not the property Πx x is a set and the sole member of x is red. But might it not be possible to understand sequences in some way that has the following feature: whatever two-or-more-term sequences may be, a one-term sequence is always identical with its first (and only) term? If one-term sequences indeed have that feature, then a one-term sequence is simply an object (in the logical sense: everything is a one-term sequence), and 'the first term of the one-term sequence x' is logically equivalent to 'x'. It would follow that 'Πx x is a one-term sequence and the first term of x is red' is—more or less—equivalent to 'Πx x is an object and x is red'. And that a theory treats that name and 'Πx x is red' as names for the same property would not seem to be objectionable on that account. Nevertheless, the idea of a theory of sequences that treats everything as a one-term sequence is at least *prima facie* incoherent. The reason is simple: if everything is a one-term sequence, then, for every n, every n-term sequence is a one-term sequence! Can something be *both* a one-term sequence and a two-term sequence? Or might it be possible to regard 'one-term sequence' as ambiguous—so that, e.g., <Denmark, Italy> is a one-term sequence in one sense of 'one-term sequence' and *not* a one-term sequence in another sense of 'one-term sequence'?

It is better to avoid having to answer these puzzling questions. We should concede that if we decide to eliminate relations in favor of relational stand-ins, we must abandon anything *resembling* the idea, any *analogue* of the idea, that properties are monadic relations. If we eliminate relations in favor of stand-ins, then our theory should have the following feature. The theory eliminates dyadic relations in favor of dyadic stand-ins, triadic relations in favor of triadic stand-ins, and so on. But it has no use for the idea of monadic stand-ins, since there is nothing for them to stand in *for*. In applying the theory, we make use of such properties as Πx x is a two-term sequence and the first term of x is to the north of the second term of x; we concede that properties like Πx (x is a one-term sequence and the first term of x is red) *exist*—so we believe if we believe that properties abound—, and are certainly in some sense "similar" to the properties of the former sort, but we have no reason to pay any attention to them.[24]

The proposal under consideration has only one disadvantage: It requires those who adopt it to have two sorts of abstract object in their ontology, properties and sequences (or properties and sets, since sequences can be understood set-theoretically). But is this a serious disadvantage? If an ontology contains properties, many philosophers will not regard the fact that it also contains sets or *sui generis* sequences as a further strike against it. And those who don't mind having properties in their ontology but don't want that ontology to contain *two* sorts of abstract object might seem to have the following option open to them: to eliminate sets and/or sequences from their ontology by some variant on Russell's "no-class" technique. If one has properties in one's ontology, why should one not "analyze sequences away" in the fashion illustrated by this example: replace

> The first member of <Denmark, Italy> is to the north of the second member of <Denmark, Italy>.

with something along these lines:

> Every property co-extensive with the following property
>
> > Being either Denmark or the property of being Italy
>
> has exactly two instances, one of which is not a property and one of which is a property; its instance that is not a property is to the north of the single instance of its instance that is a property.

After all, every workable ontology is going to have at some point to find a way to do the work sequences do. Standard mathematics and mathematical logic (the elementary theory of discrete probabilities, for example, and model theory) require either sequences or some other device that will do the work commonly assigned to sequences.

I do not think that this last option is really an option. It is certainly true that in many cases (apparent) reference to and quantification over sequences can be eliminated from one's discourse by the no-class technique. But not in all cases. Consider the sentence

> Πx (x is a two-term sequence and the first term of x is to the north of the second term of x) is not identical with Πx x is a two-term sequence and the second term of x is to the north of the first term of x.

I do not see how to use the no-class technique to eliminate the general term 'sequence' from expressions of this sort—from expressions in which that word occurs within the name of a property. (To take a simpler case, how could one use the no-class technique to eliminate reference to sets from the sentence, 'The property of being a three-membered set is not identical with the property of being a two-membered set'?) It seems, therefore, that the proposal we are considering has this feature: it forces those who adopt it to have two different and irreducible sorts of abstract object, properties and sequences, in their ontology, owing to the fact that it requires the term 'sequence' to occur within property-names (within the canonical names of stand-ins).[25]

I regard this feature of the proposal that relations be eliminated in favor of stand-ins as unfortunate, for I should like to suppose that there is only one kind of abstract object. But I recommend the proposal to philosophers who do not mind having two sorts of abstract object in their ontologies.

5. Non-symmetrical relations

At the beginning of this essay I said that the difficulties I have failed to resolve the course of my attempt to find names for proper relations are due to an assumption, an assumption that several authors have rejected. In the simplest case, the case of dyadic relations, this assumption comes to this: there are non-symmetrical dyadic relations. In this section, I present some intuitive considerations that favor the existence of non-symmetrical relations.

The account of properties that I presented in “A Theory of Properties” treats propositions as things that can be said (that is, asserted or affirmed) full stop, and properties as things that can be said *of something*. For example, “that it is white” is a thing that can be said of something (truly of the Taj Mahal, falsely of the sun). “That it is white” is, in my understanding of the word, a property—no doubt the same property that is more commonly called ‘whiteness’.

Here is an obvious feature of things that can be said of something: if one person says one of them of a certain object, and another person simultaneously says the same thing of the same object (perhaps one says, “The Taj Mahal is white” and the other says “The most famous building in Agra is white”), their two assertions must have the same truth-value.

There are things that can be said of things that cannot be said of *a* thing—that cannot be said of one thing. There are things that can be said of two things (and of no other number of things[26]), things that can be said of three things (and of no other number of things) . . . . Here, for example, is something that can be said of two things: “that the two of them live in the same city.”[27] If a thing that can be said of *a* thing is a property, then it seems reasonable to say that a thing that can be said of two things is a dyadic or binary relation. Thus propositions, properties, and proper relations are things of the same general sort. In “A Theory of Properties” (*p.* 132 *ff*) I said that propositions and properties were both “assertibles,” and I said that propositions were saturated assertibles and that properties unsaturated assertibles. We may now say that properties are singly unsaturated assertibles, that dyadic relations are doubly unsaturated assertibles, that triadic relations are triply unsaturated assertibles . . . .

The dyadic relation “that the two of them live in the same city” is like a property in the following important respect: if one person says it of two particular people, and another person says it (simultaneously) of the same two people, their two assertions must have the same truth-value. But not all dyadic relations share this feature of properties. Suppose Tom says, “Harry loves Sally” and Tim says, “Frank loves Winifred.” It would seem that there is something, some one thing, that can be said of two things (and of no other number of things) and that Tom has said it of Harry and Sally and Tim has said it of Frank and Winifred. Such a thing, a thing that can be said of *two* things is, we have said, is a dyadic relation. But suppose that a third speaker, Tam, says, “Sally loves Harry.” If Tom has said something of Harry and Sally and Tim has said it—the same thing—of Frank and Winifred, then it seems that Tam must have said it of Sally and Harry. But

Sally and Harry *are* Harry and Sally. Therefore, Tam said of Harry and Sally what Tom said of Harry and Sally. And yet—such is love—it may well be that what Tom said was true and what Tam said was false.

The dyadic relations that are like properties in that if two people say them of the same two objects what they say must have the same truth-value—"that the two of them live in the same city" was our example—we may call symmetrical relations. Dyadic relations that do not share this feature of properties we may call non-symmetrical relations. It is not easy to name them—other than by circumlocutions like 'The thing Tom said (truly) of Harry and Sally and Tam said (falsely) of Harry and Sally, and Tim said of Frank and Winifred'. They have no names of the sort exemplified by "that the two of them live in the same city." What name of that sort could we give to the thing that Tom said (etc.)? We can't call it '"that they are in love"' or '"that one of them loves the other"', for these are names of symmetrical relations.

And yet there *are* such things. There *is* something such that Tom said it of two people and Tam said it of two people (the same two) and Tim said it of two people (two other people). It is a thing that can be said of the same two people in two different "ways," and it may be true of them when said in one of these ways and false of them when said in the other. Tom said it of Harry and Sally in one of the two ways by uttering the sentence "Harry loves Sally" and Tam said it of them in the other one by uttering "Sally loves Harry." It is not a predicate or any other linguistic item—any more than a property is a linguistic item. (After all, you can say it of Harry and Sally by uttering 'Harry aime Sally' and 'Sally aime Harry'—or even 'Henricus Saram amat' and 'Sara Henricum amat'.) This thing, moreover, is a thing that is said of *two* things and not of one ordered pair. (I don't know of any argument for that conclusion, but it seems to me that what you say of two people when you say that one of them loves the other in no way involves any abstract entity of which the two people are constituents. After all, when you say that Harry and Sally live in the same city, you're saying of *them* that *they* live in the same city; you're not saying of {Harry, Sally} that *its* members live in the same city.) And if there are such things, they are non-symmetrical dyadic relations. They raise two puzzles—two puzzles that do not arise simply from their being abstract objects, two puzzles that are not also puzzles about propositions and properties and symmetrical dyadic relations. These puzzles are in fact the puzzles that we found no satisfactory solution to in Sections 2 and 3.

The first puzzle ("the puzzle of names for non-symmetrical relations") is this. I can refer to one of them only by referring to a predicate that expresses it. I can call a proposition 'the proposition that the Taj Mahal is white'; I do not have to call it 'the proposition expressed by "The Taj Mahal' is white"'. I can call a property 'the property of being (an x such that x is) white' or "that it is white"; I do not have to call it 'the property expressed by "x is white"' or 'what we say of thing when we apply the predicate "...is white" to it'. I can call a symmetrical relation 'what we say of two people when we say that they live in the same city'; I do not have to call it 'the relation expressed by "x and *y* live in the same city"' or 'what one says of two people when one applies the predicate "…and…live in the same city" to them'. But I can refer to a certain non-symmetrical relation only by using some phrase like 'what one says of two people when one applies the predicate "…loves…" to them'. (I can refer to a converse of this relation as 'what one says of two people when one applies the predicate "…is loved by…" to them'.) One might, of course, call the former relation '"loving"' and the latter '"being loved by"'. But these phrases are clear only insofar as they are abbreviations for the phrases I have just mentioned, the phrases that refer to English predicates.

And the second puzzle ("the puzzle of relation sentences") is like unto it. We are unable to refer to either of the two "ways" in which two objects can enter into a non-symmetrical relation—other than *via* reference to the predicates that express the relation. Suppose we know what relation "loving" is. Still, we have no clear way to understand 'Harry and Sally enter into/stand in/are related by "loving"'. No sentence formed from 'enter into' or 'stand in' or 'are related by' has the same truth conditions as either 'Harry loves Sally' or 'Sally loves Harry'.[28] And this puzzle has no counterpart in the case of propositions, properties, or symmetrical relations, for we understand 'The proposition that Tom lives in New York is true'; we understand 'One of the things that is true of Tom is that he lives in New York'; we understand 'One of the things that is true of Tom and Tim is that that the two of them live in the same city'.[29]

Notes

1. *Oxford Studies in Metaphysics*, Vol. 1, Dean Zimmerman (ed.), (Oxford: Oxford University Press, 2004) pp. 107-138.
2. *P*. 131. The quotation is partly from the text and partly from a footnote.
3. See Timothy Williamson, "Converse Relations," *The Philosophical Review* 94 1985, pp. 249-262; Kit Fine, "Neutral Relations," *The Philosophical Review* 109 2000, pp. 1-33; Cian Dorr, "Non-symmetric Relations," *Oxford Studies in Metaphysics*, Vol. 1, Dean Zimmerman (ed.), (Oxford: Oxford University Press, 2004), pp. 155-192.
4. I do not see why a relation should be supposed to have "a" converse—just one converse. If R_2 is a converse of R_1, and if R_3 is co-extensive with R_2 (or, better, necessarily co-extensive), is R_3 not a converse of R_1? The thesis that a relation has "a" converse is true only if necessarily co-extensive relations are *ipso facto* identical.
5. I will not discuss "multigrade relations" in this essay. For a brief discussion of multigrade relations and "variably polyadic predicates," see my book *Material Beings* (Ithaca: Cornell University Press, 1990), p. 28.
6. What does 'expresses' mean? The only definition I know of is this: an English sentence expresses a given proposition if and only the result of writing 'the proposition that' and then that sentence denotes that proposition. An excellent definition for many purposes—but not for ours.

7. More generally: The non-vacuous application of a quantifier takes a sentence in which *n* variables are free and makes a sentence in which *n* – 1 variables are free. The non-vacuous application of the operator we are looking for would take a sentence in which *n* variables were free and make a term in which *n* – 1 variables were free.
8. If 'Π*x*' is followed by a sentence in which '*x*' is not free, then it denotes the property of being such that the proposition expressed by that sentence is true.
9. This is loose speaking. An open sentence in which one variable is free expresses (if all goes well) a property, one particular property, but an open sentence in which *n* variables ($n > 1$) are free will not, in every case, express one particular relation. Suppose, for example, that '*x* loves *y*' expresses a certain relation R_1— and expresses no other relation. Then, presumably, '*y* loves *x*', would express a distinct relation R_2, a converse of R. But which relation, R_1 or R_2, would be the one expressed by '*z* loves *w*'?
10. Of course, something needs to be said about the meaning of sentences like 'Π*yz* Socrates is mortal', 'Π*yz* x is mortal', 'Π*yz* *z* is mortal', 'Π*yz* *z* loves *y* more than *y* loves *y* or *x* loves *z*' . . . .
11. Which demonstrates the point made in note 9: some sentences in which two or more variables are free do not express exactly one relation.
12. Or 'the relation such that *y* has it to *z* if and only if'. *Cf*. the definition of 'converse' in the first sentence of Williamson's "Converse Relations."
13. The suggestion does not solve the problem posed by distinct, co-extensive relations: if any relation satisfies the condition 'Denmark bears *x* to Italy if and only if Denmark is to the north of Italy', no doubt more than one does; and it may well be that more than one relation satisfies the condition '*x* is a relation such that, necessarily, Denmark bears *x* to Italy if and only if Denmark is to the north of Italy'.
14. I concede that I have allowed a simpler kind of meaningless proliferation in the case of property-names: 'Π*y* *y* is wise' and 'Π*z* *z* is wise' are pointless variants on the same property-name. But variables are needed in an expression only in cases in which there are at least two of them. In expressions that, like property-names, involve a single variable, that variable can always be eliminated in favor of 'it'. We could, if we wished, use the following form for canonical names of properties: 'The property of being something such that (. . . it . . .)'.
15. Thus, the sentence

    For any predicate P, **"**R(P)**"** is a canonical relation-name

    abbreviates the sentence

    For any predicate P, the expression that consists of 'R(' followed by P followed by ')' is a canonical relation-name;

    that is to say (for the particularly use-and-mention challenged),

    For any predicate, the expression that consists of 'R(' followed by that predicate followed by ')' is a canonical relation-name.

16. A dyadic-relation-name is a phrase that purports to denote a dyadic relation—as opposed to a dyadic name that purports to denote a relation (whatever that might mean).

17. Consider, for example, 'Brutus **and** the famous Roman orator who denounced Catiline **and** Caesar enter into . . . ' and 'Brutus **and** the famous Roman orator who denounced Catiline and Caesar enter into . . . '.
18. I do not mean to imply that a relation sentence and its non-relational counterpart in any sense say the same thing or express the same proposition. The proposition expressed by a relation-sentence and the proposition expressed by its non-relational counterpart are not even (in every case) true in the same possible worlds. Consider the case of properties. (I can't make this point for the case of proper relations, since I have no idea what proposition is expressed by any 2-or-more-term relation sentence.) Our semantics tells us that the non-relational counterpart of 'Socrates has $\Pi x \sim \exists y\ y = x$' is '$\sim\exists y\ y =$ Socrates'. And it tells us that they have the same truth-value—which is to its credit, since they're both false. But the proposition expressed by the former is *necessarily* false and the proposition expressed by the latter is only contingently false (it is true in those possible worlds in which Socrates does not exist).
19. Our conventional replacement for this sentence is 'the property of being an x such that it is not the case that x enter into x'. Strictly speaking our informal conventions do not provide a replacement for 'x enter into x' since in this sentence 'enter into' is followed by a variable and not by a canonical relation-name. But if we extend our convention in the obvious way—and replace 'x enter into x' with 'x has x'—, then 'R ($\sim$**1** enter into **1**)' may be replaced by 'the property of being an x such that it is not the case that x has x'.
20. Which, I concede, does not look very non-relational.
21. The relation R(**3** is the same color as **1** and **2**) is indifferent to permutation. For example, x, y, and z enter into this relation if and only if x, y, and z enter into R(**2** is the same color as **1** and **3**).
22. Why do I say 'one canonical name of'? It is quite plausible to suppose that 'Πx x is a two-term sequence and the first term of x is to the south of the second term of x' names the same property—the same stand-in—as 'Πx x is a two-term sequence and the second term of x is to the north of the first term of x'—and that, therefore, a stand-in can have more than one canonical name.
23. Not all properties of n-term sequences are n-adic stand-ins. Self-identity and the property of being a sequence, for example, are not dyadic stand-ins. One might make a case for the thesis that every property that entails Πx (x is an ordered pair) is a dyadic stand-in (that every property that entails Πx (x is an ordered triple) is a triadic stand-in . . .).
24. And what does the proposed theory say about propositions, since—denying, as it does, that there are relations—it cannot treat propositions as 0-adic relations? Like n-adic relations, $n > 1$, 0-adic relations may be eliminated in favor of properties (now thought of as *sui generis*, and not as a special kind of relation): the stand-in for the proposition that p will be the property of being an x that is such that p; a stand-in is *true* if something has it and *false* if nothing has it.
25. One will not face any corresponding difficulty if one supposes that there are no abstract objects but relations (properties being monadic relations)—for that view does not require any term denoting some particular ontological category—some term like 'sequence'—to appear within the formal, systematic names of relations.

26. A multigrade relation (see *n.* 5) like "that they live in the same city" can be said of two things; but it can also be said of three things, or four things, ...
27. I am not being very precise here. The word 'two' has a merely syntactical role in these formulations. Thus, I count "that the two of them are identical" as something that can be said, and said truly, of Tully and Cicero—who are, of course, not two things but one. The same point applies to "that the two of them are orators": I count saying, "Tully and Cicero are orators" as saying "that the two of them are orators" of Tully and Cicero, and saying "Demosthenes and Cicero are orators" as saying the very same thing of Demosthenes and Cicero. And I do not count either of these things as a thing that can be said of *a* thing, despite the truth of 'Cicero is identical with Tully' and 'Cicero and Tully are orators'.
28. As we have seen, sentences formed from '. . . bears . . . to . . . ' may not share this defect. It seems at least plausible to say that 'Harry bears "loving" to Sally' has the same truth-conditions as 'Harry loves Sally' and that 'Sally bears "loving" to Harry' has the same truth-conditions as 'Sally loves Harry'. If "bears" sentences indeed have this property, it is obviously due to the asymmetry created by the fact that in such sentences one term of the relation is the subject of the verb 'bears' and the other is its indirect object. But subject and indirect object are *two* grammatical categories, and there is no third category that can be used to create a form of words that stands to triadic relations as '. . . bears . . . to . . . ' stands to dyadic relations. (The category "direct object" is already taken: the relation is the direct object of 'bears'.) Therefore, even if "the problem of relation sentences" can be solved for the case of dyadic relations by writing two-term relation sentences in the form 't_1 bears r_2 to t_2', this will not—as we saw in Section 2—solve the problem for the case of triadic relations and relations of higher adicity.
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