Some Remarks on the Modal Ontological Argument

Peter van Inwagen

Abstract: This paper examines the so-called modal ontological argument. It pays special attention to the role that the symmetry and transitivity of the accessibility relation play in the argument, and examines various approaches to a defense of the "possibility premise," the premise of the argument that states that the existence of a perfect being is metaphysically possible. It contains an analysis of Gödel's attempt to show that this premise is true, and of a recent formulation by David Johnson of Gödel's argument.

When one is examining a problematical argument that involves modality, it is almost always profitable to express the modal aspect of this argument in terms of possible worlds. This is not because quantificational phrases like 'in all possible worlds' or 'in some possible word' are somehow more basic than the corresponding structureless modal operators ('necessarily,' 'possibly'). That thesis is entirely meaningless. Both devices for expressing modal propositions are firmly rooted in our ordinary thinking (consider phrases like 'No way!' and 'no matter what had happened' on the one hand, and phrases like 'couldn't be' and 'has to be' on the other). It's simply that, when one is engaged in constructing or evaluating some complicated or dubious piece of modal reasoning, it's both harder for one to make logical mistakes and easier for one to see what is going on, to grasp the argument as a whole (as opposed to simply a series of steps each of which one can see to be valid), if one uses the device of quantification over possible worlds as one's primary way of expressing modal theses.

Let me remind you of some basic ideas.[1] There are all these *possible worlds*, ways things might be. One of them is *actual*: among all the ways things might be it is the one and the only one that is the way things are. (I

Peter van Inwagen, Department of Philosophy, University of Notre Dame

don't pretend that this is a real definition of 'actual'; I'm taking 'actual' as a primitive notion.) A proposition is *true in* (*false in*) a world if it would be true (false) if that world were actual. (If we had taken truth-in-a-world rather than actuality as primitive, we could have defined actuality thus: a world is actual if and only if, for every proposition, that proposition is true in that world just in the case that it's true.) A proposition is *necessarily true* (or *necessary*) if it's true in all worlds, *possibly true* (or *possible*) if it's true in some, *necessarily false* (or *impossible*) if it's true in none, *contingently true* if it's true but not necessarily true, and *contingently false* if it's false but not impossible. An object is said to *exist in* a given possible world if it would exist if that world were actual, and to *have* a given property *in* a given possible world if it would have that property if that world were actual. An object is *contingently existent* if it exists in some but not all possible worlds and necessarily existent if it exists in all possible worlds. (We might go on to offer the following definitions: a contingently non-existent object is an object that does not exist but exists in some possible worlds ; a necessarily non-existent—or impossible—object is an object that exists in no possible world. In my view, however, these definitions are useless because nothing can satisfy them: contingently and necessarily non-existent objects are, of course, non-existent objects, and, in my view, there are and can be no non-existent objects. Those who do not share my view, however, are welcome to make what use they can of these definitions.) An object, finally, has a given property *essentially* if it has that property in every possible world in which it exists. The essential possession of a property is opposed to the *accidental* (or *contingent*) possession of a property: a thing has a property accidentally if it has it but lacks it in some world in which it exists.

The reader will note that in my usage the world 'actual' applies only to possible worlds. I have no idea what philosophers (other than David Lewis, who is a very special case) mean when they talk of "non-actual" objects generally—unless they mean non-existent objects, and, as I have said, I don't think there are or could be any of those. (What, after all, could a non-actual pig be but a non-existent pig?—and, as you may have noticed, the number of non-existent pigs is 0.) Note also that all non-actual worlds exist (not much of an accomplishment: everything exists), so it can't be that in my usage, 'actual' means 'exists'. And here is one more thing to note: the actual world is not the universe or the cosmos. The actual world is the way things are, and the universe (if we understand the universe to comprise not only the physical world, not only the furniture of earth, but the choir of heaven as well) is the things that are that way.

Our brief review ends with one final modal idea, the idea of *accessibility* or *relative possibility*. There are, as every graduate student knows, distinct, non-equivalent systems of modal logic, systems that give different answers to the question, "Which modal inferences are valid?" One way of understanding the differences between these systems turns on the notion of one possible world's being accessible from another or its being possible relative to another. We say that w_1 is *accessible from* w (or that w_1 is possible *relative to*

w) if every proposition that is true in w_1 is possible in w. We shall be interested in only one modal system, S5. S5, you will remember, is the system of modal logic whose characteristic axiom is '$\Diamond p. \rightarrow \Box \Diamond p$'. Equivalently (and this is all we need to say) S5 is the system of modal logic generated by two assumptions about the accessibility or relative-possibility relation:

> If w is accessible from w_1, then w_1 is accessible from w (the accessibility relation is symmetrical)
>
> If w is accessible from w_1, and w_1 is accessible from w_2, then w is accessible from w_2 (the accessibility relation is transitive).

Since in all modal systems that are "modal" in any but a very abstract, formal sense, the accessibility relation is reflexive (a world is always possible relative to itself), we may regard these two assumptions as equivalent to the assumption that the accessibility relation is an equivalence relation.

Let us, so armed, turn to the ontological argument.

A *modal ontological argument* is any instance of the following schema ("the Argument Schema"):

> $\Diamond \exists x$ (x exists necessarily & x has F essentially)
> *therefore*,
>
> $\exists x$ (x exists necessarily & x has F essentially).

All modal ontological arguments are valid in the modal system S5. This may be shown as follows. Assume—this is where S5 comes in—that the accessibility relation is symmetrical and transitive. We deduce the conclusion of the Argument Schema from its premise. If it is possible for there to be something that exists necessarily and has F essentially, then in some possible world w, accessible from the actual world (the AW), there is something that (in w) exists necessarily and has F essentially. Because the accessibility relation is symmetrical, the AW is accessible from w. Therefore, there exist in the AW objects that are in w necessarily existent. (That is to say, such objects actually exist, exist full stop, exist simpliciter, exist *period*.) Each of these objects is either necessary (necessarily existent) or contingent (contingently existent). Suppose one of them, x, is contingent. Then there is a possible world w_1, accessible from the AW, such that x does not exist in w_1. But the accessibility relation is transitive and w_1 is therefore accessible from w. Hence, x does not exist in a world accessible from w, and x is not necessarily existent in w, contrary to what has been assumed. Hence, there exists (actually, full stop, simpliciter, period) a necessarily existent thing.

Now suppose that x is not essentially F—that x is F only contingently or accidentally (or is not F at all). Then there is a possible world w_2, accessible from the AW, such that x exists in w_2 but is not F in w_2. Hence (because the accessibility relation is transitive) x exists but is not F in a world accessible from w, and x is not essentially F in w, contrary to what has been assumed. Hence, there exists (actually, period) a necessarily existent thing that is essentially F.

The Argument Schema is therefore valid in S5. That is to say, all modal ontological arguments are valid in S5 (and they are valid in no weaker modal system: the above proof depends essentially on the assumption that the accessibility relation is both symmetrical and transitive, and any proof of the same conclusion will require these two assumptions or something equivalent to them).

Having shown that the Argument Schema is valid, let us examine some of its instances. Suppose we substitute for ‘F’ the property-name ‘being a perfect island’ or ‘being the best possible island.’ Since the Argument Schema is valid, the argument that results from this substitution is valid. If its premise is true, therefore, there is an island that is the best possible island—and is in fact *essentially* the best possible island. Before anyone decides to mount an expedition to find this remarkable island, however, I should point out that the premise of the argument is not true. For one thing, there could not be a necessarily existent island. Furthermore, there is (or so I should suppose) no maximal degree of goodness that could belong to an island (for any possible world w in which there are islands, there is no doubt another possible world in which there’s an island better than the best island in w). And if, *per impossibile*, there were in some possible world an island that was as good as an island could be, it would not enjoy that degree of goodness essentially: it will be a less good, or even a bad, island in other possible worlds. The “perfect island” argument, therefore, is valid but unsound.

If we substitute for ‘F’ the property-name ‘concrescence’ or ‘being a concrete object’ (concrete as opposed to abstract), we obtain what I have called the Minimal Modal Ontological Argument (MMOA).[2] Presumably any concrete object is essentially concrete. If so, the MMOA is equivalent to this argument:

$\Diamond \exists x$ (x exists necessarily & x is concrete)
therefore,

$\exists x$ (x exists necessarily & x is concrete).

If we understand a necessary *being* as a necessarily existent concrete object, the validity of the MMOA is equivalent to the necessary truth of the following conditional: If it is possible for there to be a necessary being, there is (is actually, is in fact) a necessary being. The premise of the MMOA is not, like the premise of the “perfect island” argument, obviously false. But it’s not obviously true, either. The premise of the argument is equivalent to this: necessity (necessary existence) and concrescence (being a concrete object) are compatible properties. And it is not easy to see how to show either that necessity and concrescence are compatible or that they are incompatible.

If we substitute for ‘F’ in the Argument Schema a property-name that denotes the conjunction of the traditional “perfections” of God (omniscience, moral perfection, and so on), we obtain an argument (valid, of course) that we may call the Leibnizian Ontological Argument (LOA). Since necessity is itself one of the divine perfections, and since (if the accessibility relation is transitive) it is impossible for anything to have necessity as an accidental property,[3] LOA is equivalent to the following argument:

> $\Diamond\, \exists x\, x$ has every perfection essentially
> *therefore*,
>
> $\exists x\, x$ has every perfection essentially.

It is important to realize that the formal validity of the LOA in no way depends on the membership of the list of perfections or on any conceptual or metaphysical consideration pertaining to the notion of a perfection. The LOA is valid for exactly the same reason as the reason for which the "perfect island" argument is valid, this reason being the reason displayed in the proof of the validity of the Argument Schema. If we understand by "a perfect being" a being that has every perfection essentially, the validity of the LOA is equivalent to the necessary truth of the following conditional: If it is possible for there to be a perfect being, there is (is actually, is in fact) a perfect being. (Descartes defined a perfect being—the phrase he uses is 'supremely perfect being'—as a being that possesses all perfections. But it seems obvious that he ought to have said 'possesses all perfections essentially'. Consider an example. Suppose wisdom is a perfection. Consider two beings, one of which is essentially wise and the other of which is wise only accidentally. The wisdom of the former is a consequence of its nature, and the wisdom of the latter a consequence either of some cause external to its nature or else of sheer chance. It is evident that although the former may, for all we have said, be—speaking pre-analytically, our thoughts being guided not by any formal definition but only by such intuitions as we may bring to a consideration of the concept of a perfect being—a perfect being, the latter is certainly *not* a perfect being, since it is in one respect further from perfection than the former. I note that we can retain the wording of Descartes's definition if we assume—and it does seem plausible to assume this—that, for any perfection, the property of having that perfection essentially, its "essentialization," exists and is a perfection.)

What we said of the premise of the MMOA, we may say of the premise of the LOA: it's not obviously false, but it's not obviously true, either. The premise of the argument is equivalent to this: essential perfection (the property of having all perfections essentially) is a possible property. And it is not easy to see how to show either that this property is possible or that it is impossible. Indeed, it follows from our earlier statement about the premise of the MMOA that it is not easy to see how to show that essential perfection is possible: all the divine perfections other than necessity entail concrescence, and, therefore, concrescence is consistent with necessity if essential perfection is possible. It might, of course, be easy, or at least possible, to show that essential perfection was impossible even though it was not easy, perhaps not even humanly possible, to show that necessity and concrescence were incompatible. If, for example, one could show that omnipotence was an impossible property (there do exist arguments for this conclusion), that would show that essential perfection was impossible and would have no tendency to show that necessity and concrescence were incompatible. But I know of no arguments for this conclusion that have any real plausibility.

Might there be a plausible argument whose conclusion was either the premise of the LOA or its denial?

J. N. Findlay has presented an argument for the conclusion that necessity is an impossible property.[4] If this is true, then, obviously, the premise of LOA (and of every other modal ontological argument) is false. But Findlay's argument presupposes that no existential proposition can be a necessary truth, a thesis to which pure mathematics seems to provide a bottomless well of counterexamples. Findlay offers no argument for this thesis (unless 'Modern logic shows that *p*, therefore *p*' counts as an argument), and, worse, does not even consider such obvious counterexamples to it as 'There are numbers that can be expressed in more than one way as the sum of two cubes' or 'There exist functions that are everywhere continuous and nowhere differentiable'.

Kurt Gödel has devised an argument for the conclusion that the premise of the LOA is true.[5] The argument (with a few minor modifications) is this. Necessary existence and the essentialization of each of the other divine perfections (essential omniscience, essential moral perfection . . .) are all positive properties, and any set of positive properties is consistent or possible. (Necessary existence is its own essentialization or is logically equivalent to it. This was shown in *n.* 3. We could therefore express the first premise of Gödel's argument in these words: The essentialization of a perfection is always a positive property. Or, if the essentialization of a perfection is always itself a perfection, in these: Every perfection is a positive property.) Gödel's second premise, that any set of positive properties is consistent, is a consequence of two "axioms":

> The set of all positive properties is closed under entailment
> If a property is positive, its negation (complement) is not positive.

(A set of properties *entails* a given property if it is impossible for something to have all the properties in that set and to lack that property. A set of properties is *closed under entailment* if it contains every property entailed by any of its subsets.)

The proof is as follows. Suppose that the set of all positive properties is impossible or inconsistent. We show that this entails a contradiction. The set of all properties is obviously both impossible and closed under entailment.[6] Since an impossible set of properties entails any property, the *only* set of properties that is both impossible and closed under entailment is the set of all properties: the set of all positive properties is the set of all properties. But the negation of a positive property is not a positive property: the set of all positive properties is not the set of all properties. The set of all positive properties is therefore possible. It follows that any set of positive properties is possible.

But what does this proof come to? What does its conclusion *mean*? That, of course, depends on what 'positive property' means, and, unfortunately, Gödel's attempts to explain the meaning of "positive property" are compressed and cryptic. They leave the reader with no reason to suppose that there is a set of properties such that (1) necessary existence and the essentializations of the other divine perfections are members of that set, and (2) membership in that set is closed under entailment, and (3) if a property is member of that set, its negation is not. If we substitute for (1) the weaker statement 'Necessary existence and the essentializations of the members of some theologically or metaphysically interesting set of properties are members of that set', it still seems true that Gödel has provided no reason to suppose that there is a set satisfying all three requirements. Gödel's two "axioms" in fact jointly constitute a very *demanding* requirement on the set of positive properties—a fact that is demonstrated by the deducibility of the consistency of this set from the two axioms. (This statement is not meant to imply that Gödel's argument begs the question or assumes the point at issue: he can hardly be criticized for having presented an argument whose conclusion follows from its premises.) Gödel's "consistency proof" must be regarded as entirely without force until there has been added to it an account of the concept "positive property" according to which is reasonably evident that the set of positive properties satisfies conditions (1), (2), and (3)—or at least (2), (3) and *some* "membership thesis," a membership thesis that is perhaps weaker than (1) but still strong enough to be interesting. Until that has been done, his argument has no more force than the following parallel "proof" of the consistency of the axioms of first-order Zermelo-Fraenkel set-theory: Those axioms are all "positive sentences"; the set of all positive sentences is closed under logical deduction (that's an axiom of the Theory of Positive Sentences); if a sentence is a positive sentence, its negation is not (a second axiom of the Theory of Positive Sentences); hence, ZF is consistent.

One philosopher, David Johnson, has made an attempt to say what 'positive property' might be taken to mean in Gödel's argument, and has contended that the argument (when 'positive property' is interpreted in the way he suggests) has considerable force.[7] Like me, Johnson has made some changes in the way Gödel's argument is formulated. It will be convenient to employ his formulation of the argument in our discussion of his suggestion about what positive properties are. (Well, actually, I'll employ my formulation of his formulation.)

First, some definitions. Say that a second-order property (a property of properties) *descends by entailment* in just this case: if it belongs to a property, then, necessarily, it belongs to any property that property entails. (A property F entails a property G if it is impossible for something to have F and lack G.) For example, possibility descends by entailment: if a property is possible, then any property it entails must be possible. Or consider ubiquity or "being instantiated all over the place." The property of occupying space is ubiquitous; dishonesty would be a metaphysically rather less cautious example. Ubiquity descends by entailment. Consider dishonesty. If dishonesty is ubiquitous, then any property it entails is ubiquitous, for the entailed property will be instantiated in at least those places in which dishonesty is instantiated. But not every second-order property descends by entailment. For example, uninstantiation does not. The property of being a golden mountain is notoriously uninstantiated but "being made of gold" and "being a mountain" are not uninstantiated.

Here, now, is (my formulation of) Johnson's formulation of Gödel's argument. Consider the property "being an Anselmian God" or Anselmian divinity. (Anselmian divinity may be identified with the property of being a perfect being in my statement of the LOA.) Consider the second-order property "being a morally or aesthetically wonderful property, with no morally or aesthetically negative aspect." This is Johnson's reading of "being a positive property." (There is some textual justification for the hypothesis that this is something like what Gödel had in mind by "positive property."[8]) Now the word "positive" is such an abstract, milk-and-water word that, if we use it in laying out the argument, we shall perhaps find it difficult to keep its stipulated meaning in mind. In what follows, I will call this second-order property 'splendor'; instead of 'positive property' I will say 'splendid property'. We argue as follows:

> Anselmian divinity is a splendid property
>
> Splendor descends by entailment
>
> An impossible property entails every property
>
> There are properties that are not splendid (e.g., being a grain of sand, being morally evil, being both round and square).
>
> *therefore*,
>
> Anselmian divinity is a possible property.

This is Johnson's formulation of Gödel's argument—in my formulation. (I think you will be able to see that, leaving aside Johnson's reading of 'positive property', it does not differ from my formulation in any logically or philosophically fundamental way. It is, by the way, easy to see how an argument of this general sort might have suggested itself to a logician. One way to show that a set of axioms is consistent is to show, for some property, that it belongs to each of the axioms, that it is preserved by valid deduction, and that it is not a property of every well-formed formula.) Unfortunately, the second premise of this argument is obviously false, for splendor does not descend by entailment—not at any rate if there *are* any splendid properties. This is shown by the fact that every property, splendid or not, entails trivially universal properties like self-identity and being either red or not red, and trivially universal properties are pretty clearly not splendid properties. (Johnson is a very good philosopher; he is in fact a splendid philosopher. How could he have missed this obvious point? Well, even Homer nods.)

But let us not be quick to dismiss Johnson's argument. It has a false premise, but perhaps the basic idea behind the argument can be rescued. After all, to make use of the "basic idea behind the argument" we need only find *some* second-order property that (i) belongs to Anselmian divinity, (ii) descends by entailment, and (iii) does not belong to every property. The existence of any property that satisfies these three conditions will demonstrate that Anselmian divinity is a possible property. Splendor fails to satisfy the second condition, but perhaps some other property satisfies all three.

One possibility is suggested by the second part of Johnson's phrase 'being a morally or aesthetically wonderful property, with no morally or aesthetically negative aspect'. Consider the second-order property "having no morally or aesthetically negative aspect." Call this property chastity. It would seem that chastity is a property of Anselmian divinity. And it would seem not to be a property of every property—moral evil, for example, is not chaste, nor is deformity. But does chastity descend by entailment? Well, we cannot show that it *doesn't* descend by entailment by an appeal to the case of trivially universal properties, for trivially universal properties have no morally or aesthetically negative aspect. But this does not prove that chastity descends by entailment, for there are second-order properties that belong to all trivially universal properties (and to various more demanding properties as well) and which do not descend by entailment. Consider, for example, the second-order property "belonging either to nothing or to everything" ("extensional extremity," we might call it). Extensional extremity is a property of every universal property (and hence of every trivially universal property) and of every uninstantiated property. But every uninstantiated property entails properties that do not exhibit extensional extremity; for example, mermaidenhood entails maidenhood and "being a golden mountain" entails "being a mountain."

If it has not been proved that chastity descends by entailment, neither has it been proved that it does not. Let us, therefore, consider the following Johnson-style argument for the possibility of Anselmian divinity:

Anselmian divinity is a chaste property

Chastity descends by entailment

An impossible property entails every property

There are unchaste properties (e.g., moral evil and deformity).

therefore,

Anselmian divinity is a possible property.

The crucial premises of this argument are the first and the second. We have seen that it is not self-evident that the second premise is true. Perhaps we can do more than point out that the second premise is not self-evidently true, however. Perhaps we can show that it's false. Consider some uncontroversially impossible property; "being a prime number greater than all other prime numbers," for example. It would seem that this property has no morally or aesthetically negative aspect. But it entails moral evil and deformity: it is impossible for something to be the greatest prime and not to be both morally evil and deformed. Therefore, chastity does not descend by entailment.

There is, however, an obvious reply to this argument. It can be put like this.

> But 'being the greatest prime' *does* have various morally and aesthetically negative aspects. As you yourself have pointed out, it entails such unchaste properties as moral evil and deformity. And how can we say of a property that entails moral evil and deformity that it has "no morally or aesthetically

> negative aspect"? If a property F entails a property G, does the fact that F entails G not count as an "aspect" of F? If a property entails moral evil, can the fact that that entailment holds not be cited as a "negative aspect" of that property? Should this consideration not lead us to say that no impossible property is a chaste property?

Well, perhaps so. But if so, if every impossible property is an unchaste property, we cannot know whether the first premise of the argument is true unless we know whether Anselmian divinity is a possible property. It seems, therefore, that either the second premise of the argument is false, or, if it is true, then we cannot know whether the first premise of the argument is true without first determining whether its conclusion is true. And an argument with these features cannot be used to establish its conclusion.

We have not, therefore, found a second-order property that is known to belong to Anselmian divinity, known to descend by entailment, and known not to be a property of every property. We have not, as I shall say, found a second-order property that is known to be *Johnsonian*. I know of no Johnsonian property or of any property that is a plausible candidate for the office "Johnsonian property". Every candidate for this office that I have considered and examined must (I find) be rejected for reasons that are the same as, or trivial variations on, the reasons that led us to reject splendor and chastity. (This statement requires qualification. Being a theist, I believe that Anselmian divinity is a possible property. The second-order property possibility descends by entailment and is not a property of every property. If, therefore, Anselmian divinity is, as I believe, a possible property, possibility is a Johnsonian second-order property. What I mean to say is: I can see no reason that is independent of my conviction that Anselmian divinity is a possible property to believe of any second-order property that it is Johnsonian.)

This leaves us where we were before we turned to Gödel and Johnson for help: Even if there in fact is a perfect being or Anselmian God, LOA cannot serve as a means by which an inquirer can pass from *not knowing whether* there is a perfect being to *knowing that* there is a perfect being. And our position would seem to be the same with respect to MMOA. There is no reason to believe that there is a second-order property that belongs to "being a necessary being," descends by entailment, and is not a property of every property. (With this qualification: if one had some reason to believe that it was possible for there to be a necessary being, this reason would be a reason to believe that the second-order property possibility had these three features.) Therefore, MMOA cannot serve as a means by which an inquirer can pass from not knowing whether there is a necessary being to knowing that there is a necessary being.

NOTES

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1. My development of these "basic ideas" follows Alvin Plantinga's. See chapters

IV through VIII of his book *The Nature of Necessity* (Oxford: Clarendon Press, 1974).

2. See Peter van Inwagen, "Ontological Arguments" *Noûs* 11 (1977) pp. 375–395. (reprinted in Peter van Inwagen, *God, Knowledge, and Mystery: Essays in Philosophical Theology* (Ithaca, NY: Cornell University Press, 1995), pp. 22–41); Chapter 6 of Peter van Inwagen *Metaphysics* (Boulder, CO and London: Westview Press and Oxford University Press, second edition 2002).

3. *Proof* Suppose it is possible. Then in some world w, there is an x that exists in all worlds accessible from w and is only contingently existent in some world $w1$ that is accessible from w. But then there is some world w_2 accessible from $w1$ in which x does not exist. Since the accessibility relation is transitive, however, w_2 is accessible from w and x is not necessarily existent in w, which contradicts our assumption.

4. J. N. Findlay, "Can God's Existence be Disproved?," *Mind* 57 (1948), *pp*. 108–118.

5. Gödel's argument is presented in a note that was not published during his lifetime (and which was obviously not intended for publication), although he had discussed its content with various people. See "Ontological Proof" in Solomon Feferman, John W. Dawson, Jr., Warren Goldfarb, Charles Parsons, and Robert N. Solovay eds. *Kurt Gödel: Collected Works, Vol. III (Unpublished Essays and Lectures)* (Oxford: Oxford University Press, 1995), *pp*. 403–404.

6. Might the idea of a set of all properties not lead to paradoxes parallel to those that attend the idea of a set of all sets? (I should point out that *Gödel's* formulation of the argument makes no appeal to sets of properties. The problem discussed in this note is entirely an artifact of my way of presenting his argument.) This idea does in fact lead to paradox on various not unreasonable assumptions about the existence of properties (e.g., that for every set of properties there exists the property of being that set). To meet this difficulty, it suffices to point out that the argument in the text need not have been formulated in terms of sets of properties. It could instead have been formulated using "plural quantifiers" that bind "plural variables" ranging over properties. (See Peter van Inwagen, *Material Beings* (Ithaca, NY: Cornell University Press, 1990), *pp*. 23–28.) The sentences in the text that involve quantification over sets of properties could be replaced with the following sentences without affecting the point or cogency of the argument in which they occur:

> For any *x*s, if the *x*s are properties, then the *x*s entail a given property if it is impossible for anything to have all the *x*s and to lack that property.
>
> For any *x*s, if the *x*s are properties then the *x*s are closed under entailment if, for any *y*s, if the *y*s are among the *x*s, and the *y*s entail the property *z*, then *z* is one of the *x*s.
>
> For any *x*s, if something is one of the *x*s iff it is a positive property, the *x*s are closed under entailment.
>
> For any *x*s, if everything that is one of the *x*s is a positive property, then the *x*s are compossible (jointly possible, such that it is possible for there to be something that has all of them).

7. See David Johnson, *Hume, Holism, and Miracles* (Ithaca, NY: Cornell University Press, 1999), *pp*. 98–100; *Truth without Paradox* (Lanham, MD: Roman and Littlefield, 2004), pp. 118–123.

8. "Positive means positive in the moral aesthetic sense (independently of the accidental structure of the world) . . . ." Feferman *et al*. eds., p. 404.