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REVIEWS

Bigelow, John, *The Reality of Numbers: A Physicalist's Philosophy of Mathematics*, Oxford, Clarendon Press, 1988, pp. viii, 193, $65.00.

Many physicalists nowadays, and Bigelow for one, stand ready to carry metaphysical baggage when they find it worth the weight. This physicalist's philosophy of mathematics is premised on selective, *a posteriori* realism about immanent universals. Bigelow's universals, like D. M. Armstrong's, are recurrent elements of the physical world; and mathematical objects are universals. The result is a thoroughgoing threefold realism: mathematical realism, scientific realism, and the realism that stands opposed to nominalism.

Part I of the book introduces and defends the notion of *recurrent* objects: things present many times over. Properties are the obvious example, but it is relations that will be our main concern. We can't locate them exactly; but if R is a relation that relates a to b and also c to d, we can at least say that R is present within the region occupied by a and b, and also within the region occupied by c and d. It's not that R is divided between these regions; rather the whole of R is present in every region wherein there are two R-related things.

Why believe in universals?—Just because it seems that we have seen things recur in the world, and we have no good reason not to take such appearances at face value. Not for fancier theoretical reasons. Not, in particular, because positing universals would satisfy the demand that all truths must have 'truthmakers'. Part III takes up this question. (It is loosely related to the rest of the book; and unlike the rest, it is suited only to specialist readers.) Bigelow argues at length, and convincingly, that what's true cannot supervene just on which recurrent objects and ordinary particulars do and do not exist. If the universal Whiteness and the fox Albina coexist, that does not yet make it true that Albina is white. A whiteness whose very existence would entail that Albina was white could not be recurrent Whiteness. It would have to be Albina's own particular whitness, not identical to the exactly similar whitenesses of other white things. And it would have to be what C. B. Martin calls *non-transferable*: a particular whiteness that could not possibly have existed without being Albina's.

Part II, the core of the book, gives Bigelow's analysis of mathematical objects as universals. First, natural numbers. They can be used to count the (minimal) parts of a whole, the members of a set, or the instances of a property; but all of these look like special cases of something more general. Therefore Bigelow will not identify the number three, say, with the property of being tripartite, or of being three-membered, or of being three-instanced. Instead, it is a *relation*: the relation of threefold difference, we may call it. It holds between x, y, and z iff $x \neq y$, $y \neq z$, and $x \neq z$. So when we write

$$\exists x \exists y \exists z\, (Fx\ \&\ Fy\ \&\ Fz\ \&\ x \neq y\ \&\ y \neq z\ \&\ x \neq z)$$

to mean that there are at least three F's, we are not after all leaving the number three out of the story. The '$x \neq y\ \&\ y \neq z\ \&\ x \neq z$' says that x, y, and z instantiate the relational universal which is the number three.

Then the rest of the numbers: negative, rational, real, and complex. Bigelow treats all of these alike as *proportions*: relations between relations. (Here he follows in the footsteps of Eudoxus and others.) The proportions may be characterised in terms of the natural numbers, already introduced, by counting the steps in relational chains. Begin with negative and rational proportions, as follows. The parent-of relation P stands in a proportion of -1 to the offspring-of relation O, because one P-step undoes one O-step: xOy iff yPx. So -1 is the relation which, for instance, P bears to O. P stands in a proportion of 1/2 to the grandparent-of relation G, because two P-steps have the same effect as one G-step: $\exists x(yPxPz)$ iff yGz. So 1/2 is the relation which, for instance, P bears to G. Likewise 2—the proportion 2, to be distinguished from the natural number 2—is the relation which, for instance, G bears to P. Two O-steps undo one G-step, so -1/2 is the relation which, for instance, O bears to G. The great-grandparent-of relation GG stands in a proportion of 3/2 to G, because there is a relation, namely P, such that three P-steps have the effect of one GG-step, while two P-steps have the effect of one G-step. So 3/2 is the proportion which, for instance, GG bears to G. These same proportions relate other pairs of relations: -1 relates the dollar-richer relation to the dollar-poorer relation, 1/2 relates the more-by-a-pint relation to the more-by-a-quart relation, and so on.

The case of irrational proportions, such as $\sqrt{2}$, is similar. Say that we have a certain right isosceles triangle with hypotenuse h and side s; let H and S be the longer-by-h relation and the longer-by-s relation. Then $\sqrt{2}$ is the relation which, for instance, H bears to S. But this time it is harder to characterise the proportion by counting steps of chains. For no relation X can we say that a certain number of X-steps always has the effect of one H-step while another number of X-steps always has the effect of one S-step; the relations H and S, like h and s themselves, are incommensurable. But what we *can* do, so to speak, is to measure with a sequence of somewhat elastic rods. There is a relation x_1 (the longer-by-at-most-half-of-s relation) such that one X_1-step never has the effect of one S-step but two X_1-steps sometimes do, and such that two X_1-steps never have the effect of one H-step but three X_1-steps sometimes do. There is another relation X_2 such that three X_2-steps never have the effect of one S-step but four X_2-steps sometimes do, and such that five X_2-steps never have the effect of one H-step but six X_2-steps sometimes do. There is yet another relation X_3 . . . In this way we characterise the irrational $\sqrt{2}$ by a converging sequence of approximations: rational proportions, which in turn are characterised as before by counting steps.

Bigelow takes imaginary and complex proportions also as relations between relations. Let the earth be flat; then $\sqrt{2}-1$ is the 'proportion' which, for instance, relates the mile-north-of relation to the mile-east-of relation, and the mile-west-of relation in turn to the mile-north-of relation.

Bigelow is no friend of set-theoretical reductions of either sort of numbers; but to give us the rest of the objects of mathematics, he is content to give us the hierarchy of sets. Sets too are universals, recurrent objects; but queer ones, because they do not recur. Unit sets are haecceities: Bigelow's unit set is the property of being Bigelow. Other sets are 'plural haecceities'. These are relations; for instance, the set of Bigelow and Pargetter is the relation which holds beteween two things iff one of them is Bigelow and the other is Pargetter. The set of all natural numbers is the infinitary relation which holds between some things iff they are all and only the natural numbers.

Comments. First, I note that Bigelow neglects the infinite numbers. (But given the sets, they could be introduced in the usual ways.) He already asked us to grant that infinite sets are infinitary relations. Then why not say that infinite cardinals are infinitary difference relations, of a kind with the finite natural numbers? And why not say that the infinite and infinitesimal numbers of non-standard analysis are proportions, instantiated for sure somewhere in the set-theoretical hierarchy, and perhaps elsewhere as well?

Second, I complain that Bigelow's introduction to recurrence is misleading. He asks us to 'loosen up our thinking' about location in space and time, and grant that things may be multiply located. Right. But afterward it turns out—and without much warning—that what he really has in mind is something else. Spatiotemporal recurrence is just a special case. Example. Bigelow is located in a certain space-time region. His haecceity *h* is located there too, and so are *h*'s haecceity *hh* and *hh*'s haecceity *hhh*. Bigelow, *h*, and *hh* stand in the threefold difference relation which is the number three. So do *h*, *hh*, and *hhh*. Three is a universal, and here we have it instantiated twice over. It recurs. But this recurrence has nothing to do with multiple location in space and time, since all the *dramatis personae* are located in the very same region. In fact, if the world were zero-dimensional (and Bigelow were point-sized) there would be no room for any recurrence of multiple *location*, but still the recurrence of multiple *instantiation* would be going strong. Loosening up our thinking about location is all very well, but it cannot replace the vexed notion of instantiation.

Third, I complain that Bigelow's treatment requires universals to exclude and necessitate one another, yet a combinatorial approach to possibility requires them not to. If *R* stands to *S* in the proportion 1/2, for instance, then *R* cannot also stand to *S* in the proportion 1/4. Why not? If these proportions were classes of the pairs that are their instances, we could just say that 1/2 and 1/4 were classes that did not intersect. If they were 'structured meanings' abstracted from predicate phrases, we could just say that one of them contradicts the other. But if they are two different recurrent objects, as Bigelow says they are, then it is a mystery why they cannot recur in whatever combinations they please. What magic would stop them? It would be too bad if we could not say that alternative possibilities, some of them anyway, are just all the rearrangements of the elements of actuality.

So when all's said and done, we may well hesitate to accept the theory Bigelow offers us. Yet my complaints are far from fatal; his position is very much a live option. I know of no alternative—including set-theoretical orthodoxy—that is in better shape overall.

This is what philosophy ought to be: a grand vision combined with original and careful work on the details. It is presented with lucidity and modesty and good humour, and without bedazzling technicalities. An admirable book.*

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* *Errata*. Page 78. Figure 25 is badly garbled. Ignore it, the text is enough to explain the point in question. Page 132: lines 8-7 up should read ' . . entails that (N-) is false.'

Smith, Joseph Wayne, *The Progress and Rationality of Philosophy as a Cognitive Enterprise*, Aldershot, Gower, 1988, pp. vi, 301. $57.

In this book Smith sets himself a worthwhile goal: to show that philosophy is a rational enterprise which provides and makes progress. This is to be done in the face of the problem posed by the fact of perennial philosophical disagreements (PPPD), and the challenges of metaphilosophical nihilism (MN), irrationalism (MI), scepticism (MS), relativism (MR) and anarchism (MA).

Negatively, he replies to PPPD by arguing that it has not been shown that either consensus or convergence to consensus is necessary for rationality or progress; he argues that MN, MI, MS, MR and MA have not been established, and that various other attempts to solve PPPD (e.g. 'paraconsistentist' claims that philosophers asserting A and $\neg A$ may both be correct) do not succeed. Positively, he claims to show that