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Semantic analyses for dyadic deontic logic

1. INTRODUCTION

It ought not to be that you are robbed. *A fortiori*, it ought not to be that you are robbed and then helped. But you ought to be helped, given that you have been robbed. The robbing excludes the best possibilities that might otherwise have been actualized, and the helping is needed in order to actualize the best of those that remain. Among the possible worlds marred by the robbing, the best of a bad lot are some of those where the robbing is followed by helping.

In this paper, I am concerned with semantic analyses for dyadic deontic logic that embody the idea just sketched. Four such are known to me: the treatments in Bengt Hansson [4], Sections 10–15; in Dagfinn Føllesdal and Risto Hilpinen [2], Section 9; in Bas van Fraassen [9]; and in my own [8], Section 5.1.¹ My purpose here is to place these four treatments within a systematic array of alternatives,

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¹ Some other treatments of dyadic deontic logic fall outside the scope of this paper because they seem, on examination, to be based on ideas quite unlike the one I wish to consider. In particular, see the discussion in [4], [2], and [9] of several systems proposed by von Wright and by Rescher.

and thereby to facilitate comparison. There are superficial differences galore; there are also some serious differences.

My results here are mostly implicit in [8], and to some extent also in [7]. But those works are devoted primarily to the study of counterfactual conditionals. The results about dyadic deontic logic that can be extracted thence *via* an imperfect formal analogy between the two subjects are here isolated, consolidated, and restated in more customary terms.

II. LANGUAGE

The language of dyadic deontic logic is built up from the following vocabulary: (1) a fixed set of sentence letters; (2) the usual truth-functional connectives \top , \perp , \sim , $\&$, \vee , \supset , and \equiv (the first two being zero-adic ‘connectives’); and (3) the two dyadic deontic operators $O(-/-)$ and $P(-/-)$, which we may read as ‘*It ought to be that . . . , given that . . .*’ and ‘*It is permissible that . . . , given that . . .*’, respectively. They are meant to be interdefinable as follows: either $P(A/B) =_{df} \sim O(\sim A/B)$ or else $O(A/B) =_{df} \sim P(\sim A/B)$. Any sentence in which $O(-/-)$ or $P(-/-)$ occurs is a *deontic sentence*; a sentence is *iterative* iff it has a subsentence of the form $O(A/B)$ or $P(A/B)$ where A or B is already a deontic sentence. (We regard a sentence as one of its own subsentences.) In metalinguistic discourse, as exemplified above, vocabulary items are used to name themselves; the letters early in the alphabet, perhaps subscripted, are used as variables over sentences; and concatenation is represented by concatenation.

III. INTERPRETATIONS

$\llbracket \]$ is an *interpretation* of this language *over* a set I iff (1) $\llbracket \]$ is a function that assigns to each sentence A a subset $\llbracket A \rrbracket$ of I , and (2) $\llbracket \]$ obeys the following conditions of standardness:

- (2.1) $\llbracket \top \rrbracket = I$,
- (2.2) $\llbracket \perp \rrbracket = \emptyset$,
- (2.3) $\llbracket \sim A \rrbracket = I - \llbracket A \rrbracket$,
- (2.4) $\llbracket A \& B \rrbracket = \llbracket A \rrbracket \cap \llbracket B \rrbracket$,

- (2.5) $\llbracket A \vee B \rrbracket = \llbracket A \rrbracket \cup \llbracket B \rrbracket$,
(2.6) $\llbracket A \supset B \rrbracket = \llbracket \sim A \vee B \rrbracket$,
(2.7) $\llbracket A \equiv B \rrbracket = \llbracket (A \supset B) \& (B \supset A) \rrbracket$
(2.8) $\llbracket P(A/B) \rrbracket = \llbracket \sim O(\sim A/B) \rrbracket$.

We call $\llbracket A \rrbracket$ the *truth set* of a sentence A , and we say that A is *true* or *false* at a member i of I (under the interpretation $\llbracket \rrbracket$) according as i does or does not belong to the truth set $\llbracket A \rrbracket$.

We have foremost in mind the case that I is the set of all possible worlds (and we shall take the liberty of calling the members of I *worlds* whether they are or not). Then we can think of $\llbracket A \rrbracket$ also as the proposition expressed by the sentence A (under $\llbracket \rrbracket$): an interpretation pairs sentences with propositions, a proposition is identified with the set of worlds where it is true, and a sentence is true or false according as it expresses a true or false proposition.

The sentences of the language are built up from the sentence letters by means of the truth-functional connectives and the deontic operators. Likewise an interpretation is determined stepwise from the truth sets of the sentence letters by means of the truth conditions for those connectives and operators. (2.1–7) impose the standard truth conditions for the former. (2.8) transforms truth conditions for $O(-/-)$ into truth conditions for $P(-/-)$, making the two interdefinable as we intended. The truth conditions for $O(-/-)$ have so far been left entirely unconstrained.

IV. VALUE STRUCTURES

Our intended truth conditions for $O(-/-)$ are to depend on a posited structure of evaluations of possible worlds. We seek generality, wherefore we say nothing in particular about the nature, source, or justifiability of these evaluations. Rather, our concern is with their structure. A mere division of worlds into the ideal and the less-than-ideal will not meet our needs. We must use more complicated value structures that somehow bear information about comparisons or gradations of value.

An interpretation is *based*, at a particular world, on a value structure iff the truth or falsity of every sentence of the form $O(A/B)$, at that

world and under that interpretation, depends in the proper way on the evaluations represented by the value structure.

Let $\llbracket \cdot \rrbracket$ be an interpretation over a set I , and let i be some particular world in I . In the case we have foremost in mind, I really is the set of all possible worlds; and i is our actual world, so that truth at i is actual truth, or truth *simpliciter*. We consider value structures of four kinds.

First, a *choice function* f over I is a function that assigns to each subset X of I a subset fX of X , subject to two conditions: (1) if X is a subset of Y and fX is nonempty, then fY also is nonempty; and (2) if X is a subset of Y and X overlaps fY , then $fX = X \cap fY$. $\llbracket \cdot \rrbracket$ is *based, at i , on* a choice function f over I iff any sentence of the form $O(A/B)$ is true at i under $\llbracket \cdot \rrbracket$ iff $f\llbracket B \rrbracket$ is a nonempty subset of $\llbracket A \rrbracket$. Motivation: fX is to be the set of the best worlds in X . Then $O(A/B)$ is true iff, non-vacuously, A holds throughout the B -worlds chosen as best.

Second, a *ranking* $\langle K, R \rangle$ over I is a pair such that (1) K is a subset of I ; and (2) R is a weak ordering of K . R is a *weak ordering*, also called a *total preordering*, of a set K iff (1) R is a dyadic relation among members of K ; (2) R is transitive; and (3) for any j and k in K , either jRk or kRj – that is, R is *strongly connected* on K . $\llbracket \cdot \rrbracket$ is *based, at i , on* a ranking $\langle K, R \rangle$ over I iff any sentence of the form $O(A/B)$ is true at i under $\llbracket \cdot \rrbracket$ iff, for some j in $\llbracket A \& B \rrbracket \cap K$, there is no k in $\llbracket \sim A \& B \rrbracket \cap K$ such that kRj . Motivation: K is to be the set of worlds that can be evaluated – perhaps some cannot be – and kRj is to mean that k is at least as good as j . Then $O(A/B)$ is true iff some B -world where A holds is ranked above all B -worlds where A does not hold.

Third, a *nesting* $\$$ over I is a set of subsets of I such that, whenever S and T both belong to $\$$, either S is a subset of T or T is a subset of S . $\llbracket \cdot \rrbracket$ is *based, at i , on* a nesting $\$$ over I iff any sentence of the form $O(A/B)$ is true at i under $\llbracket \cdot \rrbracket$ iff, for some S in $\$$, $S \cap \llbracket B \rrbracket$ is a nonempty subset of $\llbracket A \rrbracket$. Motivation: each S in $\$$ is to represent one permissible way to divide the worlds into the ideal ones (those in S) and the non-ideal ones. Different members of $\$$ represent more or less stringent ways to draw the line. Then $O(A/B)$ is true iff there is some permissible way to divide the worlds on which, non-vacuously, A holds at all ideal B -worlds.

Fourth, an *indirect ranking* $\langle V, R, \uparrow \rangle$ over I is a triple such that (1) V

is a set; (2) R is a weak ordering of V (defined as before); and (3) \hat{f} is a function that assigns to each j in I a subset $\hat{f}(j)$ of V . $\llbracket \cdot \rrbracket$ is *based, at i , on* an indirect ranking $\langle V, R, \hat{f} \rangle$ iff any sentence of the form $O(A/B)$ is true at i under $\llbracket \cdot \rrbracket$ iff, for some v in some $\hat{f}(j)$ such that j belongs to $\llbracket A \ \& \ B \rrbracket$, there is no w , in any $\hat{f}(k)$ such that k belongs to $\llbracket \sim A \ \& \ B \rrbracket$, such that wRv . Motivation (first version): V is to be a set of ‘values’ realizable at worlds; wRv is to mean that w is at least as good as v ; and $\hat{f}(j)$ is to be the set of values realized at the world j . Then $O(A/B)$ is true iff some value realized at some B -world where A holds is ranked higher than any value realized at any B -world where A does not hold. Motivation (second version): we want a ranking of worlds in which a single world can recur at more than one position – much as Grover Cleveland has two positions in the list of American presidents, being the 22nd and also the 24th. Such a ‘multipositional’ ranking cannot be a genuine ordering in the usual mathematical sense, but we can represent it by taking a genuine ordering R of an arbitrarily chosen set V of ‘positions’ and providing a function \hat{f} to assign a set of positions – one, many, or none – to each of the objects being ranked. Then $O(A/B)$ is true iff some B -world where A holds, in some one of its positions, is ranked above all B -worlds where A does not hold, in all of their positions.

The *value structures over I* comprise all four kinds: all choice functions, rankings, nestings, and indirect rankings over I . Note that (unless I is empty) nothing is a value structure of two different kinds over I .

An arbitrary element in our truth conditions must be noted. A value structure may ignore certain *inevaluable* worlds: for a choice function f , the worlds that belong to no fX ; for a ranking $\langle K, R \rangle$, the worlds left out of K ; for a nesting $\$$, the worlds that belong to no S in $\$$; and for an indirect ranking $\langle V, R, \hat{f} \rangle$, the worlds j such that $\hat{f}(j)$ is empty. Suppose now that B is true only at some of these inevaluable worlds, or that B is impossible and true at no worlds at all. Then $O(-/B)$ and $P(-/B)$ are *vacuous*. We have chosen always to make $O(A/B)$ false and $P(A/B)$ true in case of vacuity, but we could just as well have made $O(A/B)$ true and $P(A/B)$ false. Which is right? Given that $0 = 1$, ought nothing or everything to be the case? Is everything or nothing permissible? The mind boggles. As for formal elegance,

either choice makes complications that the other avoids. As for precedent, van Fraassen has gone our way but Hansson and Føllesdal and Hilpinen have gone the other way. In any case, the choice is not irrevocable either way. Let $O'(-/-)$ and $P'(-/-)$ be just like our pair $O(-/-)$ and $P(-/-)$ except that they take the opposite truth values in case of vacuity. The pairs are interdefinable: either let $O'(A/B) =^{\text{df}} O(\top/B) \supset O(A/B)$ or else let $O(A/B) =^{\text{df}} \sim O'(\perp/B) \& O'(A/B)$.

V. TRIVIAL, NORMAL, AND UNIVERSAL VALUE STRUCTURES

There exist *trivial* value structures, of all four kinds, in which every world is inevaluable. We might wish to ignore these, and use only the remaining non-trivial, or *normal*, value structures. Or we might go further and use only the *universal* value structures with no inevaluable worlds at all. It is easily shown that a value structure is normal iff, under any interpretation based on it at any world i , some sentence of the form $O(\top/B)$ is true at i . (And if so, then in particular $O(\top/\top)$ is true at i .) Likewise, a value structure is universal iff, under any interpretation based on it at any world i , any $O(\top/B)$ is true at i except when B is false at all worlds.

VI. LIMITED AND SEPARATIVE VALUE STRUCTURES

The *limited* value structures are, informally, those with no infinitely ascending sequences of better and better and better worlds. More precisely, they are: (1) all choice functions; (2) all rankings $\langle K, R \rangle$ such that every nonempty subset X of K has at least one *R-maximal element*, that being a world j in X such that jRk for any k in X ; (3) all nestings \mathcal{S} such that, for any nonempty subset \mathbf{S} of \mathcal{S} , the intersection $\bigcap \mathbf{S}$ of all sets in \mathbf{S} is itself a member – the smallest one – of \mathbf{S} ; and (4) all indirect rankings $\langle V, R, \mathfrak{f} \rangle$ such that, if we define the *supersphere* of any v in V as the set of all worlds j such that wRv for some w in $\mathfrak{f}(j)$, then for any nonempty set \mathbf{S} of superspheres, the intersection $\bigcap \mathbf{S}$ of all sets in \mathbf{S} is itself a member of \mathbf{S} . Clearly some but not all rankings, some but not all nestings, and some but not all indirect

rankings are limited. Value structures of any kind over finite sets, however, are always limited.

Semantically, a limited value structure is one that guarantees (except in case of vacuity) that the full story of how things ought to be, given some circumstance, is a possible story. That is not always so. For instance, let the value structure be a ranking that provides an infinite sequence j_1, j_2, \dots of better and better worlds. Let B be true at all these worlds and no others; let A_1 be true at all but j_1 , A_2 at all but j_1 and j_2 , and so on. Then $O(-/B)$ is not vacuous and all of $O(B/B)$, $O(A_1/B)$, $O(A_2/B)$, \dots are true; yet at no world are all of B, A_1, A_2, \dots true together, so even this much of the story of how things ought to be, given that B , is impossible. A limited ranking would preclude such a case, of course, since the set $\{j_1, \dots\}$ has no maximal element. In general, a value structure is limited iff, under any interpretation based on it at any world i , whenever $O(-/B)$ is non-vacuous and \mathbf{A} is the set of all sentences A for which $O(A/B)$ is true at i , there is a world where all the sentences in \mathbf{A} are true together.

The *separative* value structures are, informally, those in which any world that surpasses various of its rivals taken separately also surpasses all of them taken together. More precisely, they are: (1) all choice functions; (2) all rankings; (3) all nestings \mathcal{S} such that, for any nonempty subset \mathbf{S} of \mathcal{S} , the intersection $\bigcap \mathbf{S}$ is the union $\bigcup \mathbf{T}$ of some subset \mathbf{T} of \mathcal{S} ; and (4) all indirect rankings such that, for any nonempty set \mathbf{S} of superspheres, the intersection $\bigcap \mathbf{S}$ is the union $\bigcup \mathbf{T}$ of some set \mathbf{T} of superspheres. All limited value structures are separative, but not conversely. Some but not all non-limited nestings are separative, as are some but not all non-limited indirect rankings. Semantically, a value structure is separative iff, under any interpretation based on it at any world i , if (1) A is true at just one world, (2) $O(A/B)$ is true at i for every B in a set \mathbf{B} , and (3) C is true at just those worlds where at least one B in \mathbf{B} is true, then $O(A/C)$ is true at i .

VII. CLOSED AND LINEAR VALUE STRUCTURES

A nesting \mathcal{S} is *closed* iff, for any subset \mathbf{S} of \mathcal{S} , the union $\bigcup \mathbf{S}$ of all sets in \mathbf{S} belongs to \mathcal{S} . Closure has no semantic effect, as we shall see, but

we must mention it in order to make contact with my results in [8]. Note that a closed nesting \mathcal{S} is separative iff, for any nonempty subset \mathbf{S} of \mathcal{S} , $\cap \mathbf{S}$ belongs to \mathcal{S} .

An indirect ranking $\langle V, R, \mathfrak{f} \rangle$ is *linear* iff there are no two distinct members v and w of V such that both vRw and wRv . We shall see that linearity also has no semantic effect.

VIII. EQUIVALENCE

We call two value structures *equivalent* iff any interpretation that is based, at a world, on either one is also based, at that world, on the other. Equivalence is rightly so called: it is a reflexive, symmetric, transitive relation among value structures, and consequently it partitions them into equivalence classes. If two value structures are equivalent, they must be value structures over the same set; and if one is trivial, normal, universal, limited, or separative, then so is the other.

If f is any choice function over I , an equivalent ranking $\langle K, R \rangle$ over I may be derived thus: let K be the set of all i in I such that i is in $f\{i\}$, and let iRj (for i and j in K) iff i is in $f\{i, j\}$.

If $\langle K, R \rangle$ is any limited ranking over I , an equivalent choice function f over I may be derived thus: for any subset X of I , let fX be the set of all R -maximal elements of $X \cap K$ (and empty if $X \cap K$ is empty). Note that if the given ranking had not been limited, the derived f would not have been a genuine choice function.

If $\langle K, R \rangle$ is any ranking over I , an equivalent nesting \mathcal{S} over I may be derived thus: let \mathcal{S} contain just those subsets of K such that for no j in the subset and i outside it does iRj hold.

If \mathcal{S} is any separative nesting over I , an equivalent ranking $\langle K, R \rangle$ over I may be derived thus: let K be the union $\cup \mathcal{S}$ of all sets in \mathcal{S} , and let iRj (for i and j in K) iff there is no set in \mathcal{S} that contains j but not i . Note that if the given nesting had not been separative, the derived ranking would not have been equivalent to the nesting.

If \mathcal{S} is any nesting over I , an equivalent indirect ranking $\langle V, R, \mathfrak{f} \rangle$ over I may be derived thus: let V be \mathcal{S} , let vRw (for v and w in V) iff v is included in w , and let $\mathfrak{f}(i)$, for any i in I , be the set of all members of V that contain i .

If $\langle V, R, \mathfrak{f} \rangle$ is any indirect ranking over I , an equivalent nesting \mathcal{S}

may be derived thus: let \mathcal{S} be the set of all superspheres of members of V .

If \mathcal{S} is any nesting over I , an equivalent closed nesting \mathcal{S}' may be derived thus: let \mathcal{S}' be the set of all unions $\cup \mathbf{S}$ of subsets \mathbf{S} of \mathcal{S} .

Finally, if $\langle V, R, \mathfrak{f} \rangle$ is any indirect ranking over I , an equivalent linear indirect ranking $\langle V', R', \mathfrak{f}' \rangle$ over I , may be derived thus: let V' be a subset of V such that, for any v in V , there is exactly one w in V' such that vRw and wRv ; let R' be the restriction of R to V' ; and let $\mathfrak{f}'(i)$, for any i in I , be $\mathfrak{f}(i) \cap V'$.

We can sum up our equivalence results as follows. Say that one class of value structures is *reducible to* another iff every value structure in the first class is equivalent to one in the second class. Say that two classes are *equivalent* iff they are reducible to each other.

(1) The following classes are equivalent:

all nestings,
all indirect rankings.

(2) The following classes are equivalent; and they are reducible to the classes listed under (1), but not conversely:

all rankings,
all separative nestings,
all separative indirect rankings.

(3) The following classes are equivalent; and they are reducible to the classes listed under (2) and (1), but not conversely:

all choice functions,
all limited rankings,
all limited nestings,
all limited indirect rankings.

(4) Parts (1)–(3) still hold if we put ‘closed nesting’ throughout in place of ‘nesting’, or if we put ‘linear indirect ranking’ throughout in place of ‘indirect ranking’, or both.

(5) Parts (1)–(4) still hold if we restrict ourselves to the normal value structures of each kind, or to the universal value structures of each kind.

So the fundamental decision to be taken is not between our four kinds of value structures *per se*. Rather, it is between three levels of generality: limited, separative, and unrestricted. Once we have decided on the appropriate level of generality, we must use some class of value structures versatile enough to cover the chosen level; but it is a matter of taste which of the equivalent classes we use.

IX. FRAMES

Suppose that an interpretation is to be based, at our actual world, on a given value structure of some kind. Suppose that the truth sets of the sentence letters also are given. To what extent is the interpretation thereby determined? First, we have the truth sets of all non-deontic sentences – that is, of all truth-functional compounds of sentence letters. Second, we have the actual truth values of all non-iterative deontic sentences – that is, of all truth-functional compounds of sentences of the forms $O(A/B)$ and $P(A/B)$, where A and B are non-deontic, together perhaps with non-deontic sentences. But there we stop, for we know nothing about the truth conditions of $O(-/-)$ and $P(-/-)$ at non-actual worlds. Hence we do not have the full truth sets of the non-iterative deontic sentences. Then we do not have even the actual truth values of iterative deontic sentences. (Apart from some easy cases, as when a deontic sentence happens to be a truth-functional tautology.)

To go on, we could stipulate that the interpretation is to be based at *all* worlds on the given value structure. But that would be too rigid. Might not some ways of evaluating worlds depend on matters of fact, so that the value structure changes from one world to another? What we need, in general, is a family of value structures – one for each world. Call this a *frame*. A frame might indeed assign the same value structure to all worlds – then we call it *absolute* – but that is only a special case, suited perhaps to some but not all applications of dyadic deontic logic.

We have four kinds of frames. A *choice function frame* $\langle f_i \rangle_{i \in I}$ over a set I assigns a choice function f_i to each i in I . A *ranking frame* $\langle K_i, R_i \rangle_{i \in I}$ over I assigns a ranking $\langle K_i, R_i \rangle$ to each i in I . A *nesting frame* $\langle \mathcal{S}_i \rangle_{i \in I}$ over I assigns a nesting \mathcal{S}_i to each i in I . An *indirect ranking frame* $\langle V_i, R_i, \hat{f}_i \rangle_{i \in I}$

over I assigns an indirect ranking $\langle V_i, R_i, \mathfrak{f}_i \rangle$ to each i in I . (I ignore *mixed frames*, which would assign value structures of more than one kind.) A frame is *trivial*, *normal*, *universal*, *limited*, *separative*, *closed*, or *linear* iff every value structure that it assigns is so. An interpretation over I is *based on* a frame over I iff, for each world i in I , the interpretation is based at i on the value structure assigned to i by the frame. Given that an interpretation is to be based on a certain frame, and given the truth sets of the sentence letters, the interpretation is determined in full.

Two frames are *equivalent* iff any interpretation based on either one is based also on the other, and that is so iff both are frames over the same set I and assign equivalent value structures to every i in I . One class of frames is *reducible to* another iff every frame in the first class is equivalent to one in the second. Two classes of frames are *equivalent* iff they are reducible to each other. Then we have reducibility and equivalence results for frames that are just like the parallel results for single value structures.

X. VALIDITY

A sentence is *valid under* a particular interpretation over a set I iff it is true at every world in I ; *valid in* a frame iff it is valid under every interpretation based on that frame; and *valid in* a class of frames iff it is valid in all frames in that class. Let us consider six sets of sentences, defined semantically in terms of validity in classes of frames. The sentences in each set are just those that we would want as theorems of dyadic deontic logic if we decided to restrict ourselves to the frames in the corresponding class, so we may call each set the *logic determined by* the corresponding class of frames.

- CO:** the sentences valid in all frames.
- CD:** the sentences valid in all normal frames.
- CU:** the sentences valid in all universal frames.
- CA:** the sentences valid in all absolute frames.
- CDA:** the sentences valid in all absolute normal frames.
- CUA:** the sentences valid in all absolute universal frames.

The six logics differ: by restricting ourselves to the normal, universal, or absolute frames we validate sentences that are not valid in broader classes. But the logics do not change if, holding those restrictions fixed, we also restrict ourselves to the separative frames, the limited frames, or the frames over finite sets; or to the indirect ranking frames, linear indirect ranking frames, nesting frames, closed nesting frames, ranking frames, or choice function frames. By these latter restrictions we validate no new sentences.

For instance, take any sentence A that does not belong to the logic **CO**, not being valid in all frames. Then in particular, by our equivalence results, it is invalid under some interpretation $\llbracket \cdot \rrbracket$ based on a nesting frame $\langle \mathcal{S}_i \rangle_{i \in I}$. Now define $\langle \mathcal{S}_i^* \rangle_{i \in I^*}$ and $\llbracket \cdot \rrbracket^*$ as follows: (1) for each i in I , let D_i be a conjunction of all the subsentences or negated subsentences of A that are true (under $\llbracket \cdot \rrbracket$) at i ; (2) let I^* be a subset of I that contains exactly one world from each nonempty $\llbracket D_i \rrbracket$; (3) for any subset S of I , let $*S$ be the set of all i in I^* such that $\llbracket D_i \rrbracket$ overlaps S ; (4) for each i in I^* , let \mathcal{S}_i^* be the set of the $*S$'s for all S in \mathcal{S}_i ; and (5) let $\llbracket \cdot \rrbracket^*$ be an interpretation based on $\langle \mathcal{S}_i^* \rangle_{i \in I^*}$, which is a nesting frame, such that whenever B is a sentence letter, $\llbracket B \rrbracket^*$ is $\llbracket B \rrbracket \cap I^*$. It may then be shown (see [8], Section 6.2, for details) that whenever C is a subsentence of A , $\llbracket C \rrbracket^*$ is $\llbracket C \rrbracket \cap I^*$. Since that is so for A itself, A is invalid under $\llbracket \cdot \rrbracket^*$. Further, I^* is finite: it contains at most 2^n worlds, where n is the number of subsentences of A . So we do not validate A by restricting ourselves to the class of nesting frames over finite sets, the broader class of limited nesting frames, the still broader class of separative nesting frames, or any other class equivalent to one of these. Exactly the same proof works for the other five logics; we need only note that if $\langle \mathcal{S}_i \rangle_{i \in I}$ is normal, universal, or absolute, then so is $\langle \mathcal{S}_i^* \rangle_{i \in I^*}$.

As a corollary, we find that our six logics are decidable. The question whether a sentence A belongs to one of them reduces, as we have seen, to the question whether A is valid in the appropriately restricted class of nesting frames over sets with at most 2^n worlds, n being the number of subsentences of A ; and that is certainly a decidable question.

XI. AXIOMATICS

We may axiomatize our six logics as follows. For **CO** take the rules R1–R4 and the axiom schemata A1–A8.² For **CD** add axiom A9; for **CU** add A10 and A11; for **CA** add A12 and A13; for **CDA** add A9, A12, and A13; and for **CUA** add A10, A12, and A13.

R1. All truth-functional tautologies are theorems.

R2. If A and $A \supset B$ are theorems, so is B .

R3. If $A \equiv B$ is a theorem, so is $O(A/C) \equiv O(B/C)$.

R4. If $B \equiv C$ is a theorem, so is $O(A/B) \equiv O(A/C)$.

A1. $P(A/C) \equiv \sim O(\sim A/C)$.

A2. $O(A \& B/C) \equiv O(A/C) \& O(B/C)$.

A3. $O(A/C) \supset P(A/C)$.

A4. $O(\top/C) \supset O(C/C)$.

A5. $O(\top/C) \supset O(\top/B \vee C)$.

A6. $O(A/B) \& O(A/C) \supset O(A/B \vee C)$.

A7. $P(\perp/C) \& O(A/B \vee C) \supset O(A/B)$.

A8. $P(B/B \vee C) \& O(A/B \vee C) \supset O(A/B)$.

A9. $O(\top/\top)$.

A10. $A \supset O(\top/A)$.

A11. $O(\top/A) \supset P(\perp/P(\perp/A))$.

A12. $O(A/B) \supset P(\perp/\sim O(A/B))$.

A13. $P(A/B) \supset P(\perp/\sim P(A/B))$.

These axiom systems for **CO**, **CD**, **CU**, **CA**, **CDA**, and **CUA** have been designed to use as many as possible of the previously proposed axioms discussed in [2], [4], and [9]. To establish soundness and completeness, we need only check that our axiom systems are

² For any fixed C , we can regard $O(-/C)$ and $P(-/C)$ as a pair of monadic deontic operators. R1–R3 and A1–A3, in which the fixed C figures only as an inert index, constitute an axiom system for Lemmon’s weak deontic logic **D2** (see [5], [6]) for each such pair. **D2** falls short of the more standard deontic logic **D** for the pair by lacking the theorem $O(\top/C)$; nor should that be a theorem since it is false in case of vacuity and some instances of $O(-/C)$ are vacuous. Had we used $O'(-/-)$ and $P'(-/-)$ we would still fall short of **D**: in case of vacuity we would then have $O'(\top/C)$, but we would lose the instances of A3. Rather we would have the logic **K** for each pair, as in the basic conditional logic of Chellas [1].

equipollent to those given in [8], Section 6.1, for the ‘V-logics’ **V**, **VN**, **VTU**, **VA**, **VNA**, and **VTA**, respectively; for those logics, in a definitional extension of our present language, are known to be determined by the appropriately restricted classes of separative closed nesting frames (there called *systems of spheres*). Our **CO** and **CD** are equipollent also to their namesakes in [7] and [9], respectively.

XII. COMPARISONS AND CONTRASTS

It is an easy task now to compare the four previous treatments listed at the beginning. I include also my treatment of **CO** in [7], although **CO** is presented there only as a minimal logic for counterfactuals, without mention of its deontic reinterpretation.

A. *Hansson* [4]. (We take only the final system **DSDL3**.) Language: operators with the truth conditions of our $O'(-/-)$ and $P'(-/-)$; iteration prohibited; truth-functional compounding of deontic and non-deontic sentences also prohibited. Semantic apparatus: universal limited rankings. (The relation of these to choice functions is studied in *Hansson* [3].)

B. *Føllesdal and Hilpinen* [2]. Language: operators with the truth conditions of our $O'(-/-)$ and $P'(-/-)$; iteration not discussed. Semantic apparatus: semiformal; essentially our universal choice functions. It is suggested that the best worlds where a circumstance holds are those that most resemble perfect worlds. That improves the analogy, otherwise merely formal, with counterfactuals construed as true (as in my [7] and [8]) iff the consequent holds at the antecedent-worlds that most resemble our actual world. But I feel some doubt. Lilies that fester may smell worse than weeds, but are they also less similar to perfect lilies?

C. *Van Fraassen* [9]. Language: $O(-/-)$ and $P(-/-)$; iteration permitted. Semantic apparatus: normal linear indirect ranking frames. These are motivated in the first of our two ways: values realized at worlds are ranked, not whole worlds with all their values lumped together. The idea may be that values are too diverse to be lumped together;

but if so, are they not also too diverse to be ranked? (Van Fraassen may agree, for in [10] he has since developed a pluralistic brand of deontic logic meant to cope with clashes of incomparable values.) The need for non-separative indirect rankings does not seem to me to have been convincingly shown.

D. *Lewis* [8]. Language: operators with the truth conditions of all four of ours; iteration permitted. Semantic apparatus: separative closed nesting frames, with normality, universality, and absoluteness considered as options; ranking frames also are mentioned by way of motivation. It is argued that more than limited frames are needed, since infinite sequences of better and better worlds are a serious possibility.

E. *Lewis* [7]. Language: one operator, with the truth conditions of our $O'(-/-)$; iteration permitted. Semantic apparatus (three versions): (α) partial choice function frames; (β) nesting frames; and (γ) ranking frames.

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