Review of Charles Chihara, *The Worlds of Possibility*

(Oxford: Clarendon Press, 1998. Pp. xii, 342)

Theodore Sider *Philosophical Review* 110 (2001): 88-91

Possible worlds present a formidable challenge for the lover of desert landscapes. One cannot ignore their usefulness; they provide, as David Lewis puts it, "a philosophers' paradise".[1] But to enter paradise possibilia must be fit into a believable ontology. Some follow Lewis and accept worlds at face value, but most prefer some other choice from the current menu.

Part of Chihara's book is a critical discussion of some of these menu options: Lewis's modal realism, Alvin Plantinga's abstract modal realism, Graeme Forbes's anti-realism and Gideon Rosen's modal fictionalism. These discussions are very detailed and conversant with the literature. The discussions of Forbes (pp. 142-167) and of paradox within Plantinga's system (pp. 120-141) are particularly enlightening.

The rest of the book is devoted to Chihara's positive project: developing an account of the status of model theory for *non*-modal logic, and then applying it to the modal case. The prize is an understanding of possible worlds semantics that requires no commitment to possible worlds at all (beyond the purely formal "possible worlds" of the standard Kripke-models.)

What does the relativized notion of *truth in an interpretation* studied in (non-modal) model theory have to do with plain old truth? Chihara's answer involves "connecting theorems" that relate truth-in to truth (chapter 5). A "natural-language proto-interpretation of the sentential calculus" (NLPI of **SC**) is a function that assigns meanings of declarative sentences of English to sentence letters. Where I is an NLPI of **SC** and ϕ is a sentence letter, Chihara uses '$[\phi/I]$' to refer to "ϕ with the meaning it has been assigned by I" (p. 191); where ϕ is not atomic, he says (p. 192):

> '$[\phi/I]$' is to refer [*sic*] the sentence ϕ, when each sentential letter occurring in ϕ is to be understood to have the meaning assigned to it by I, and the logical connectives occurring in ϕ are understood in the classical truth functional way.

I *conforms* to a (standard sentential) interpretation **I** iff for every sentence letter, ϕ, ϕ is true relative to **I** iff $[\phi/I]$ is true (*simpliciter*). The connecting theorem

[1] *On the Plurality of Worlds* (Oxford: Basil Blackwell, 1986), chapter 1.

is then:[2]

> For every interpretation **I** of **SC**, every NLPI I conforming to **I**, and every sentence ϕ of **SC**, ϕ is true under **I** iff $[\phi/I]$ is true

An important corollary is:

> For every NLPI I conforming to interpretation **I**, for every set of sentences of **SC** Γ, and for every sentence ϕ of **SC**, if ϕ is a model-theoretic consequence of Γ, then, if every member θ of Γ is such that $[\theta/I]$ is true, then $[\phi/I]$ is true.

Thus, model-theoretic consequence can guide inference in English.

The theorem and corollary are supposed to relate truth-in to the ordinary notion of truth, but apply the latter notion to $[\phi/I]$, for sentences ϕ of **SC**. Truth is a property of natural language sentences or propositions; $[\phi/I]$ is neither. The introduction of the notion "$[\phi/I]$" quoted above is obscure. One reconstruction of what it means is that $[\phi/I]$ is to be true *simpliciter* iff ϕ is true in any interpretation **I** to which I conforms. That I understand, but then the theorem no longer relates truth-in-an-interpretation to the ordinary notion of truth. The same problem confronts the modal versions of the theorem discussed below.

The problem could perhaps be solved as follows. Let I assign (unambiguous) declarative sentences of English to sentence letters. Recursively define $[\phi/I]$ as a sentence of English whose structure mirrors that of ϕ: if ϕ is atomic then $[\phi/I] = I(\phi)$; if $\phi = \ulcorner(\psi \& \chi)\urcorner$ then $[\phi/I] = \ulcorner[\psi/I]$ and $[\chi/I]\urcorner$, etc. (somehow eliminating scope ambiguity.) $[\phi/I]$ will then be a sentence of English, to which the ordinary notion of truth applies. *Mutatis mutandis* for the cases of propositional and predicate modal logic.

Chihara then gives an analogous account for modal logic. The account above presupposed the boolean notions ("in the metalanguage"); the modal account similarly uses unexplained modal notions. Chihara is fully explicit here: he is not trying to analyze modality (pp. 207-209). NLPI of the modal sentential calculus (**MSC**) are the same as for **SC**. I *conforms* to an S5 interpretation (of the standard sort) $\boldsymbol{I} = \langle \boldsymbol{W}, w*, v \rangle$ of **MSC** iff (p. 213):

[i] For every w in $\boldsymbol{W}$, the world could have been as each $\boldsymbol{v}_I(\sigma, w)$ says that it is

[2] My formulations of the theorem and corollary are equivalent to those on 192-193.

[ii] It is not the case that the world could have been such that no $\boldsymbol{v}_I(\sigma,w)$ represents it as being that way

[iii] For every sentential letter ϕ, ϕ is true under $\boldsymbol{I}$ iff $[\phi/I]$ is true

The notation $\boldsymbol{v}_I(\sigma,w)$ is confusing, since Chihara never defines the symbol σ. It seems to mean the conjunction of all sentence letters of **MSC** that are true at w in $\boldsymbol{I}$ with the negations of all sentence letters false at w in $\boldsymbol{I}$, interpreted according to I, for Chihara says (p. 212):

> ...$\boldsymbol{v}_I(\sigma,w)$ can be regarded more specifically as representing how the world could have been, in so far as what are of concern are the truth values of the sentential letters, *when they are interpreted as expressing the statements assigned to them by I.* (His emphasis)

But this sentence won't exist if the language has infinitely many sentence letters. Never mind; Chihara says on p. 213 that talk of $\boldsymbol{v}_I(\sigma,w)$ is just heuristic. What, then, do [i] and [ii] really amount to? Following other remarks on p. 213, perhaps:

[i′] For every w in $\boldsymbol{W}$, it is possible that: for every sentence letter θ, $[\theta/I]$ is true iff $\boldsymbol{v}(\theta,w)=0$

[ii′] It is not possible that: there is no w in $\boldsymbol{W}$ such that for every sentence letter θ, $[\theta/I]$ is true iff $\boldsymbol{v}(\theta,w)=0$

(0 in Chihara's models indicates truth, not falsity.)

We then get the modal connecting theorem: (p. 213)

Theorem [2] For every S5 interpretation $\boldsymbol{I}=\langle \boldsymbol{W}, \boldsymbol{w*}, \boldsymbol{v}\rangle$ of **MSC**, for every NLPI I conforming to $\boldsymbol{I}$, and for every w in $\boldsymbol{W}$, a sentence ϕ is true at w under $\boldsymbol{I}$ iff $[\phi,I]$ would have been true, had the world in fact been such that, for every sentential letter θ, $[\theta,I]$ is true iff $\boldsymbol{v}(\theta,w)=0$.

As before, the theorem has corollaries that relate model-theoretic consequence to truth *simpliciter*.

At one point in the proof of this theorem[3] the following inference is made: (p. 206)

[3]The proof is not actually given, but asserted to be analogous to that of a related earlier theorem; it is that proof on pp. 204-207 that I discuss. In my discussion I convert the notation of the earlier theorem to that of the later.

[D] Possibly, $[\psi/I]$ is true

Therefore, there is some $w \in \boldsymbol{W}$, such that $[\psi/I]$ would have been true, had the world been such that $\boldsymbol{v}_I(\sigma,w)$ was true

The reasoning given appeals to "$\boldsymbol{v}_I(\sigma,w)$" and the initial formulation of [ii]: [ii] implies that for every way the world could have been, some $\boldsymbol{v}_I(\sigma,w)$ "represents it being that way"; [D] states that $[\psi/I]$ is a way the world could be; therefore some $\boldsymbol{v}_I(\sigma,w)$ represents $[\psi/I]$ as being the case; therefore, had this $\boldsymbol{v}_I(\sigma,w)$ been true, $[\psi/I]$ would have been true. But that formulation of [ii] was just loose talk ($\boldsymbol{v}_I(\sigma,w)$ need not exist; moreover talk of "ways the world could be" and "representation" should be avoided.) When we substitute [ii′] for [ii] and reword the conclusion of the inference to remove reference to $\boldsymbol{v}_I(\sigma,w)$, the inference isn't so smooth:

[D] Possibly, $[\psi/I]$ is true

Therefore, there is some $w \in \boldsymbol{W}$, such that $[\psi/I]$ would have been true, had the world been such that for every sentential letter $\theta, [\theta, I]$ is true iff $\boldsymbol{v}(\theta,w) = 0$.

Given [D], [ii′] guarantees[4] that there's some $w \in \boldsymbol{W}$ such that it's possible that $[\psi/I]$ is true and for every sentence letter $\theta, [\theta/I]$ is true iff $\boldsymbol{v}(\theta, w) = 0$. But the conclusion of the argument does not immediately follow. The gap can presumably be spanned by a simple induction: roughly, non-modal $[\psi/I]$ have their truth values decided by the $[\theta/I]s$ by the truth tables; modal $[\psi/I]$ are settled vacuously by the $[\theta/I]s$ given the S5 principle.

The analogous theorems Chihara proves for quantified modal logic are considerably more complex and ingenious. If my worries about truth *simpliciter* for sentences $[\phi/I]$ can be answered, the theorems provide an innovative and powerful account of the usefulness of model theory for modal logic.

But there is a kind of disconnect between the halves of the book. The first concerns the metaphysics of Lewis, Plantinga, Forbes and Rosen; the second logic. This is intentional; Chihara's main thesis is that you don't need metaphysically loaded possible worlds to understand modal logic. But understanding modal logic isn't all the metaphysicians were after. They want more:

[4]Assuming that the sets in S5 interpretations exist necessarily, and that membership does not vary from world to world.

i) an account of quantification over possibilities in everyday language, for example "there are five ways to win this chess match"[5]

ii) tools for providing analyses (not accounts of inference, but analyses) of counterfactuals, supervenience, chance, propositions, properties, etc.

iii) a reductive account of modality (in Lewis's case)

iv) tools for use in linguistics, for example in Montague grammar

v) an illumination of what the correct modal logic should be (Chihara needs to assume S5 to establish his theorems. Garbage in, garbage out.)

Chihara's theorems seem not to help here, not straightforwardly anyway. We still need an ontology of possibilia to enter fully into paradise.

[5] Chihara discusses this on p. 275.