MODEL-THEORY VERSUS FICTIONALISM

Putnam famously attempted to refute metaphysical realism using model-theoretic arguments. Putnam's own response to these arguments was to advocate *internal realism* in place of metaphysical realism. But might the correct response to the model-theoretic arguments sometimes be *fictionalism*? The purpose of this paper is to investigate the relationship between fictionalism and internal realism.

I shall start by briefly explaining Putnam's model-theoretic arguments against metaphysical realism (§1). Metaphysical realists think of truth in terms of reference and correspondence; the model-theoretic arguments seek to undermine this account of truth by showing that reference and correspondence relations are hopelessly underdetermined. I shall show that the model-theoretic arguments are particularly effective in attacking realism about mathematical entities, and so I shall focus mostly on fictionalism about mathematical entities.

Mathematical fictionalists treat mathematical theories as *false*, but useful, fictions (§2). If they ever say that a mathematical theory is true, they mean only that it is *true-in-the-fiction*, which is not to say that it is *really* true. Evidently fictionalists have two notions of truth; so we can attempt to raise the model-theoretic arguments in two places.

Fictionalists can deal with both arguments. But in doing so, they end up sounding very much like internal realists. I shall attempt to put clear water between fictionalism and internal realism, without much success (§3). This leads to some striking metaontological conclusions, which fictionalists and realists alike may find surprising.

1 Model-theory and metaphysical realism

In this section, I shall explain the doctrine that Putnam calls *metaphysical realism*. I shall then outline the model-theoretic arguments against metaphysical realism.

1.1 Metaphysical realism explained

A *metaphysical realist* is a kind of objects-realist: she thinks that there are objects, which are mind-, theory- and language-independent. Moreover, she believes that the truth-values of sentences or thoughts are ultimately fixed by some objective "correspondence relation between words or thought-signs and external things and sets of things."[1]

I have no wish to suggest that everyone who calls herself a realist must accept the idea that truth involves correspondence. To that extent, "metaphysical realism" is a term of art whose meaning is fixed by stipulation. But it is a useful term of art, for the doctrine of metaphysical realism has formed the backbone of an extremely influential version of realism.

[1] Putnam (1981, p. 49). Also Putnam (1982, p. 30; 1989, p. 214).

Metaphysical realists attempt to flesh out their notion of correspondence by emphasising that they are not *pragmatists*. In particular, metaphysical realists think that, no matter how 'ideal' any given theory is, that theory might be false.[2] The paradigm case is as follows. Imagine that humans have arrived at some scientific theory, which is marvellous on all imaginable pragmatic criteria: it predicts all our observations perfectly; it retrodicts and explains faultlessly; it is simple, harmonious, and beautiful; to learn the theory gives the sensation of seeing into the mind of God; *&c.* In that sense, the theory is *ideal*.[3] For the pragmatist, no more is required of the theory for it to count as true. For the metaphysical realist, this absolutely ideal theory could (in principle) be false. It might be false because some skeptical hypothesis is true (for example, we might all be brains in vats). It might be false because it gets the basic ontology of the world wrong. It might even be false for reasons we cannot even state, because we lack the language to do so. The metaphysical realist postulates an epistemological gulf between truth and ideality.

Still, the metaphysical realist has stuck her neck out, and advanced some theory of the world, T, which she hopes is true. T contains some names, "a_1", "a_2",..., and some primitive predicates, "R_1", "R_2",.... We now use model-theory to capture the metaphysical realist's doctrine that there is some definite notion of reference and correspondence between the (object) language of T and mind-independent objects. In particular, the metaphysical realist is to think of the world as the intended model, $\mathcal{W}$, of her favourite theory, T. The domain of $\mathcal{W}$ is to be thought of as the objects of the world. An interpretation function maps the names "a_1", "a_2", ... of the language of T to objects $a_1^{\mathcal{W}}, a_2^{\mathcal{W}}, \ldots$ in the domain of $\mathcal{W}$. $\mathcal{W}$ also contains sets, $R_j^{\mathcal{W}}$, which contain ordered n-tuples of urelements, and which are to be thought of the extension of each predicate "R_j" of T.

We can now think of reference—both to individuals and to properties—in terms of the interpretation of names and predicates. When the metaphysical realist says that "'a' refers to a", we can parse this as saying that the interpretation function maps "a" to $a^{\mathcal{W}}$. This also gives us an easy way to consider correspondence. Schematically, the idea is that, for atomic sentences containing a single predicate:

$$\text{“}R_j(a_1, \ldots, a_n)\text{”} \quad \text{is true iff} \quad \langle a_1^{\mathcal{W}}, \ldots, a_n^{\mathcal{W}} \rangle \, \varepsilon \, R_j^{\mathcal{W}}$$

Complex sentences containing quantifiers and sentential connectives are to be evaluated recursively, in the standard Tarskian fashion.

1.2 The model-theoretic arguments

We can now use some elementary results from model-theory to raise problems for the metaphysical realist's notion of reference and correspondence. I shall focus on two arguments, the first of which arises from a very simple theorem:

> **Permutation Theorem:** Let T be a non-trivial theory. If T has a model, $\mathcal{W}$, then there is a permuted model $\mathcal{P}$ such that:

[2] See Putnam (1978, p. 125), Putnam (1980, p. 473) and Putnam (1989, p. 214).

[3] See Putnam (1978, p. 125).

(i) $\mathcal{W}$ and $\mathcal{P}$ share the same domain
(ii) $\mathcal{W}$ and $\mathcal{P}$ are isomorphic
(iii) $\mathcal{W} \neq \mathcal{P}$.

So, if $\mathcal{W}$ models T, then there is some distinct permuted model, $\mathcal{P}$ which is isomorphic to $\mathcal{W}$. *A fortiori*, they both make exactly the same sentences of T true. But then we have a free choice as to say that "a" refers to $a^{\mathcal{W}}$ or that "a" refers to $a^{\mathcal{P}}$, and so as to whether to treat the correspondence relation as given by the base cases:

$$\text{“}R_j(a_1, \ldots, a_n)\text{”} \quad \text{is true iff} \quad \langle a_1^{\mathcal{W}}, \ldots, a_n^{\mathcal{W}} \rangle \, \varepsilon \, R_j^{\mathcal{W}}$$

as the metaphysical realist hopes, or by the base cases of the permuted model:

$$\text{“}R_j(a_1, \ldots, a_n)\text{”} \quad \text{is true iff} \quad \langle a_1^{\mathcal{P}}, \ldots, a_n^{\mathcal{P}} \rangle \, \varepsilon \, R_j^{\mathcal{P}}$$

In short, the truth-values of the sentences in T are insufficient to determine what the reference and correspondence relations "really" are. This is Putnam's permutation argument.[4]

The second argument arises from another elementary result of model-theory:

> **Skolem's Theorem:** Let T be any consistent countable set of sentences of a first-order language. Then T has a model, $\mathcal{N}$, whose domain is the natural numbers.

If the metaphysical realist presents us with a first-order theory T, whose intended interpretation $\mathcal{W}$ is uncountable, then there is an unintended countable model, $\mathcal{N}$, of T. Once again, we have free choice as to whether to treat truth, reference and correspondence as given by $\mathcal{W}$, or as given by $\mathcal{N}$. So no matter how much the metaphysical realist protests that the intended interpretation is *really* countable, if she is still speaking the object language, she cannot tell between $\mathcal{W}$ and $\mathcal{N}$. Too many models would make her theory true.

1.3 Could something else fix reference?

Both arguments seek to embarrass the metaphysical realist by showing that her twin notions of reference and correspondence are hopelessly, utterly, underdetermined by her theory.

On their own, though, Putnam's arguments only demonstrate that fixing the truth-values of every sentence in T is insufficient to fix reference and correspondence relations. Metaphysical realists can respond by saying that something *else* fixes these relations. For example, many philosophers have thought that *causation* somehow fixes reference. So a decent argument against metaphysical realism would seem to require (at least) a full appraisal of all versions of the causal theory of reference. This would be an extremely arduous task.

[4]See Putnam (1981, pp. 32–8, 217–8).

Fortunately, it is a task that we can often shirk. If the metaphysical realist thinks that the entities to which she refers are acausal, then there can be no way, even in principle, that causation could fix reference to those objects.[5] For example, metaphysical realists about mathematical entities typically believe that mathematical entities are acausal; so reference to mathematical entities cannot be fixed causally.

Could something other than causation fix reference to mathematical entities? One might follow the route of Gödelian platonism, and postulate a faculty of mathematical intuition which gives us direct access to mathematical objects. Such a faculty would allow us to refer directly to mathematical objects. But Putnam was exactly right to say that "this appeal to mysterious faculties seems both unhelpful as epistemology and unpersuasive as science."[6]

It is difficult to think of anything else that could fix reference to mathematical entities. The model-theoretic arguments therefore have particular bite against metaphysical realism about mathematical entities. Obviously, much more needs to be said to confirm this; but not in this paper. Instead, *in this paper I shall assume that the model-theoretic arguments have successfully undermined metaphysical realism.* To make this assumption plausible, and for the sake of concreteness, I shall concentrate particularly on the case of mathematics. However, it should be noted that the focus on mathematics is inessential to the argument of this paper: if the model-theoretic arguments undermine metaphysical realism in *another* domain (e.g. if they undercut metaphysical realism about the objects of contemporary physics) then the same considerations will apply.

The question is: having abandoned metaphysical realism, where do we go? Might we, in particular, turn to fictionalism?

2 Model-theory *&* fictionalism

In this section, I shall outline (mathematical) fictionalism, and attempt to subject it to the model-theoretic arguments. By the end of this section, we shall have arrived at a version of fictionalism which is immune to the model-theoretic arguments.

2.1 Fictionalism's two levels of truth

Fictionalists think that some area of discourse is to be treated, not as literally true, but as a useful fiction. Obviously, fictionalists of different walks have different understandings of what "useful" means, but the general idea is that a theory can have many virtues—simplicity, strength, explanatory or predictive success, *&c.*—without being true:

> The fictionalist's distinctive claim is that a false claim can be *ideally* accept-

[5]Causation could, of course, play a part in how reference is *transmitted* from speaker to speaker; but only *after* it has been fixed by non-causal means.

[6]Putnam (1980, p. 471).

> able. For the fictionalist, literal falsity is simply not a defect and literal truth as such is not a virtue.[7]

We should immediately recall from §1.1 that metaphysical realists also think that a false theory can be *ideally* acceptable. The key difference between fictionalists and metaphysical realists is this. Metaphysical realists think that truth is a virtue which may be absent even if the theory is perfect in every other way. Fictionalists do not think that truth is a virtue for a theory which is perfect in every other way. So metaphysical realists and fictionalists both employ the same notion of truth; the difference between them is a difference in their attitude towards truth.

Actually, that simplifies things too much: fictionalists really require *two* notions of truth.

First, they need a notion of *literal truth*. This is the notion of truth that I just mentioned, which fictionalists use when they say things like "of course, what I have been telling you is just a fiction; it isn't *really true*."

Second, they need a notion of truth *within the fiction*. Consider the following exchange between two fictionalists about mathematics:

—— $\varnothing$ has no subsets.
—— That's *false*: $\varnothing$ is a subset of itself!
—— Oh, *true*; good point.

The participants are using a notion of *truth within the fiction* (of mathematics).

Metaphysical realists and fictionalists share their notion of literal truth. Indeed, it is precisely *because* fictionalists have that notion of literal truth, and think that it does not apply to their fiction, that they can identify themselves as fictionalists at all (rather than as metaphysical realists). So the interesting question is: What is this the fictionalist's notion of truth *within* a fiction?

One might think that truth within the fiction is simply closure under deductive entailment from the text of the fiction. If so, then fictionalism about mathematics is formalism, for the "text of the fiction" is surely just the (axiomatised) mathematical theory. Formalists do not talk about reference to mathematical objects at all. So whatever formalism's failings, it certainly has nothing to fear from the model-theoretic arguments.

To keep things interesting, I shall reserve the term "fictionalism" for a position that is *not* formalism.[8] This position moves beyond fictions as *texts*, and treats fictions as what one might tentatively call "story-worlds". These are the fantastic worlds described *by* fictions. One might come to represent these by pretending that one inhabits the story-world.[9] That is: *to be a fictionalist is to act as if the fiction were true.*

Recall that the original model-theoretic arguments attacked the metaphysical realist's notion of truth. We have just seen that fictionalists have *two* notions of truth: literal truth, and truth within the fiction (understood as truth within a

[7] Rosen (2005, p. 16).

[8] As a caution to the reader: some self-described "fictionalists" are really just formalists. Wagner (1982, p. 263) is a prime example.

[9] See Walton (1990).

pretence / story-world). So, in principle there are two places in which we might attempt to raise model-theoretic arguments against fictionalism. By considering both arguments, I shall obtain Two Morals for fictionalists. Sticking to these Morals will inoculate fictionalists against the model-theoretic arguments.

2.2 The model-theoretic arguments, within the story

I shall first offer the model-theoretic arguments within the context of the fiction / pretence / story-world. The argument has the same form as it had when we raised it against metaphysical realism; we simply need to preface every sentence of the argument with "Within the context of the story…".

Within the context of the story, the fictionalist explains the intended model, $\mathcal{W}$, of her favourite mathematical theory T. Within the context of the story, we then run a model-theoretic argument against her, generating an unintended model, $\mathcal{P}$ or $\mathcal{N}$, say. This deviant model makes true (again, within the story) exactly the same sentences as $\mathcal{W}$; but with the wrong reference and correspondence relations. So, within the story, reference relations are radically underdetermined.

Fictionalists ought to respond to this argument by stating that it is just part of their fiction that reference is fixed. One way to do this is as follows. In §1.3 I insisted that we do not have a Gödelian faculty which enables us to refer to mathematical entities directly. The fictionalist should agree with me about that: of course no one *literally* has direct mental access to transfinite cardinals. But—the fictionalist can continue—nothing stops us from *pretending* to have that ability, or from acting *as if* we do. That is, our fictionalist can simply pretend to be a Gödelian platonist.

This response is flawless. However, it requires some unpacking.

Suppose the fictionalist *first* pretends that there are lots of mathematical objects—sets, numbers, *&c.*—and *then* pretends that we can refer to them directly, by postulating (within the fiction) some Gödelian platonist faculty of mathematical intuition. In that case, the model-theoretic arguments would return between the two stages. The resources required to run the arguments—namely, model-theory and a domain of objects—would be available after the first stage. For example, suppose that person A pretends that "$\varnothing$" refers determinately to $\varnothing^A$, and person B pretends that "$\varnothing$" refers determinately to $\varnothing^B$; then we would have no guarantee that, in the pretence, $\varnothing^A = \varnothing^B$. We would have no reason to think that different mathematicians were pretending to refer to the same objects.

To avoid this, fictionalists ought to maintain that there is never any question of whether (in the pretence) "$\varnothing$" refers tot $\varnothing$. This leads to our first moral for fictionalists:

> **Moral 1:** To pretend that there are objects, and to pretend that we can refer to these objects, is a single act of pretence; it must not be divided into two distinct stages.

2.3 The model-theoretic arguments, outside the story

We next try running the model-theoretic arguments *outside* the scope of the story. Recall that the fictionalist is not simply treating the fiction of mathematics merely as a text (§2.1, but is working within a story-world (within which reference is fixed, by Moral 1). We aim to ask: *which* story-world is she working within?

The original model-theoretic arguments showed us that a theory does not pick out a single model. Here, they might show us that a theory does not pick out a single story-world; rather, multiple *different* story-worlds would satisfy the story. In each of these (following Moral 1) we can allow that reference is fixed. But we can maintain that reference is fixed differently in different story-worlds. That is, according to each world, "$\varnothing$" refers to $\varnothing^{\mathcal{W}}$; but one world's $\varnothing^{\mathcal{W}}$ is another's $\varnothing^{\mathcal{P}}$ (say). And nothing the fictionalist can say will single out one, rather than the other, as *the intended* story-world.

Fictionalists ought to respond that this argument misunderstoods what "story--worlds" are. They should say that the argument treats "story-worlds" as nothing more than models. (Indeed, they should probably say that the sentence "one world's $\varnothing^{\mathcal{W}}$ is another's $\varnothing^{\mathcal{P}}$" is unintelligible.)

Again, this response is flawless; but again, it needs some unpacking.

The fictionalist has only told us, so far, that story-worlds are not like models. But she has not told us what they *are* like; nor has she told us what "generates" the *single* story-world that (on pain of mathematical solipsism) we ought to believe that all mathematicians (can) inhabit. Primitive faculties that put us in direct contact with mathematical objects are of no use here, because the fictionalist does not think that there are, literally speaking, any mathematical objects for us to be put in direct contact with.

Really, all we have to go on is our *use*—in the broadest sense—of the fiction of mathematics. Under the umbrella of use, we have the formal theories of mathematics; we have everything that goes on in mathematics classrooms; we have everything that is written informally and formally in mathematics textbooks and journals; *&c.* But we have nothing other than use. This leads to our second moral for fictionalists:

> **Moral 2:** The totality of human mathematical practice *alone* fixes the pretence / story-world.

3 Fictionalism *&* Internal Realism

Fictionalists who adhere to both Morals are inoculated against the model-theoretic arguments. But in this section, I want to consider how different such fictionalists are from Putnam himself. I shall highlight several similarities between fictionalism and internal realism, and I shall draw a Dummettian conclusion. Having rejected metaphysical realism, the interesting questions that arise concern not *ontology*, but *the nature of truth*.

3.1 Internal realists accept both Morals

Putnam's own response to the model-theoretic arguments was as follows:

> Either the use *already* fixes the 'interpretation' or *nothing* can.... [T]he world does not pick models or interpret languages. *We* interpret our languages or nothing does.... Models are not lost noumenal waifs looking for someone to name them; they are constructions within our theory itself, and they have names from birth.[10]

The very first sentence tells us that internal realists alike should believe that *use* is all that fixes interpretation. That was our fictionalist's Moral 2.

Second, Putnam denies the perspective from which the model-theory can be wielded against the problems of reference. Putnam thinks that we could never doubt that "$\varnothing$" refers to $\varnothing$, since our entry-point to the object language and the metalanguage are identical in each case. ("*We* interpret our languages or nothing does".) That was the fictionalist's Moral 1.

Accordingly, internal realists and fictionalists alike share Morals 1 and 2. The natural question that arises is: What is the difference between internal realism and fictionalism?

3.2 Different theories simultaneously?

One plausible difference is as follows. The internal realist might be inclined to treat set-theory (for example) as about *the unique hierarchy of the sets*. The fictionalist, by contrast, might countenance many different stories simultaneously about many different set-hierarchies: in one story, $\mathbf{V} = \mathbf{L}$; in another, the Axiom of Choice is false; *&c.*

If one person countenances just one hierarchy, and another person countenances many hierarchy, then the two people obviously occupy different positions! However, *this* difference does not cut along the internal realism / fictionalism axis.[11]

At one end of the spectrum, we have what we might call the "full-blooded" internal realist, who will countenance all kinds of different set-hierarchies. This position finds its counterpart with the fictionalist who is happy to deal with many different stories. At the other end of the spectrum, we have the austere internal realist, who countenances just one set-hierarchy (or maybe only a handful of them). This would correspond to the fictionalist who has taken seriously the fact that most mathematicians are only interested in a small number of set theories, and restricts himself to only participating in a few stories, treating the rest as mere formalism.

Marginally, it would probably follow convention better to use "internal realist" for more austere positions and reserve "fictionalism" for more full-blooded positions. But this conventional association is only marginal and it is a difference in degree, not in kind.

[10] Putnam (1980, p. 482).

[11] cf. Hale's (2007) comment that the austere / full-blooded distinction does not cut along the axis that supposedly distinguishes metaphysical realists from noneists.

3.3 Different theories sequentially?

We just considered whether fictionalists might accept multiple stories simultaneously. We might do better by considering which stories fictionalists might accept *sequentially*. Suppose the fictionalist first pretends that there is just one set-hierarchy and that $\mathbf{V} = \mathbf{L}$, then later pretends that there is just one set-hierarchy and that the Axiom of Choice is false. Presumably, she has *pretended* to change her opinion about the hierarchy of sets. By contrast, an internal realist who talks about different unique hierarchies at different times has surely genuinely changed her position. Surely this flags a serious difference between internal realism and fictionalism?

This apparent difference arises just from caricaturing both internal realism and fictionalism. Writing about internal realism generally, Putnam says:

> Different statements—in some cases, even statements that are 'incompatible' from the standpoint of classical logic and classical semantics—can be true in the same situation because the words—in some cases, the logical words themselves—are used differently.[12]

So Putnam thinks that, in the situation just envisaged, the internal realist need not in fact have changed her *opinion*, but only her *vocabulary*. Conversely, fictionalists would typically unhappy if they found themselves frequently flip-flopping back and forth between (pretending to believe) apparently incompatible theories. After all, the more one merrily pretends to change one's position, the less one can merrily pretend to be a metaphysical realist. In this regard, van Fraassen suggests that fictionalists as much as realists must look for a single unified physical theory to pretend to believe.[13]

Accordingly, relative freedom of movement between different theories cannot easily be used to characterise the difference between internal realism and fictionalism.

3.4 On the very idea of an external perspective

The best way to draw a distinction between fictionalism and internal realism would be as follows. Fictionalists think that, literally speaking, there are no sets, and that the symbol "$\varnothing$" does not refer to anything at all, let alone to a particular set. She thinks that set theory is not literally true. So we might be able to find a distinction between internal realism and fictionalism by asking the question: Does the internal realist think that her theory is literally true? This turns out to be a surprisingly complicated question.

In the most straightforward sense, the internal realist thinks that her theory is literally true. For she certainly thinks that her theory is true and, since she never engages in pretence, she thinks that it's literally true, and literally literally true, and literally literally literally true, *&c.* Perhaps this is the difference between

[12] Putnam (1988, pp. 115–6).

[13] Fraassen (1980, pp. 80–7). Van Fraassen's *constructive empiricism* is certainly a version of fictionalism; see Kalderon (2005, pp. 2ff.).

internal realists and fictionalists: one thinks their theory is literally true, and the other thinks that their theory is not.

Again, things are not quite so simple. On one reading, the fictionalist is using the "according to the fiction" operator merely to flag that she is not a metaphysical realist about mathematics. But the internal realist is not a metaphysical realist about mathematics either. Indeed, when talking with a metaphysical realist, she might agree that her theory is not "literally true" (scare quotes needed), because to say that her theory is "literally true" might be taken as suggesting that she was a metaphysical realist. Similarly, she might agree that the symbol "$\varnothing$" does not "literally refer" (scare quotes again), so as to make clear that she is not working with the metaphysical realist's notion of reference. So, in conversation with metaphysical realists, fictionalists and internal realists speak together.

What's more, the word "literally" is utterly insignificant in all this. When the fictionalist is caught up in the pretence, she may say that her theory is *literally* true, or that "$\varnothing$" literally refers to $\varnothing$. She would not be wrong to do so; she would only indicate a serious change of heart if she asserted *outside* of the fiction that her theory was *literally* true. But she would indicate the very same change of heart by asserting (again, outside of the fiction) simply that her theory was true. Which is to say: the qualifier "literally", once added to a language, can always be (ab)used.

Having disregarded the word "literally", we are left with the following. Internal realists and fictionalists are simply using the words "true" and "false" slightly differently. Fictionalists (when not pretending) use "true" in the same as metaphysical realists; internal realists use "true" in a different way.

This is not a very interesting difference! Indeed, as things stand, literally nothing hangs on whether we decide to follow the fictionalists' or internal realists' use. A decision either way would change nothing within the philosophy classroom: it will not flag any difference in whether or not we believe that there are "metaphysically real" mathematical objects, for both fictionalists and internal realists agree that there are not. And a decision either way will obviously change nothing *outside* the philosophy classroom: pupils will still be praised when they answer math-questions correctly; physicists will still use the same equations, with the same degree of success; people will still make exactly the same mistakes in basic arithmetic. So, as things stand, the decision cannot change anything: we simply have a *free* choice as to how to use the words "true" or "false", and so as to whether to call ourselves fictionalists or internal realists. (I shall revisit this in a moment.)

At this point, a standard move in metaphysical disputes would be to say that fictionalism wins by default. After all, it seems that fictionalism is less ontologically committing than internal realism, since it (literally speaking) countenances no mathematical objects, whereas internal realists (literally speaking) do. This default move would be exceptionally foolish. *If there is nothing but a verbal difference between fictionalism and internal realism, then there is nothing but a verbal difference between their ontological commitments.* Put conversely: to perceive a difference in the ontological commitment of fictionalists and internal realists, we would need to have *already* perceived a difference between their respective uses of "truth" that was not merely verbal.

3.5 The significance of the theory of truth

Everything, then, comes down to the theory of truth. It is worth noting that, following Peirce, Putnam took the internal realist's notion of (literal) truth to be truth in the ideal limit of inquiry. (Indeed, at one point Putnam went so far as to say "I should have called [internal realism] pragmatic realism!"[14])

I do not want to suggest that internal realists *must* follow Putnam and adopt a pragmatist theory of truth. For example, Dummettian anti-realists do not face the model-theoretic arguments; they believe that meaning is use; and they think that there "literally" are mathematical objects. So internal realists might just as easily be Dummettian anti-realists, rather than Peircean pragmatists.

How would internal realists decide either way? By considering the theory of truth, of course. The important point is that, once we have rejected metaphysical realism (perhaps because we have accepted the model-theoretic arguments), questions about the theory of truth assume paramount importance. We want to know what theory of truth we ought to use. We want to know whether that theory of truth justifies all the laws of classical logic and, if it does not justify all the laws without restriction, in which situations those laws *are* justified. If we answer these questions in such a way that our theory of truth does not vindicate all the laws of classical logic, then we shall have to request a reform of philosophical and mathematical practice. Questions about the theory of truth lead to serious decisions concerning how to proceed in the teeth of the model-theoretic arguments.

By contrast, the "decision" considered at the end of the preceding subsection is utterly trivial. It does not matter whether or not we choose to ourselves "internal realists" or "fictionalists". Likewise, it does not matter whether we decide to think that something "literally exists" or merely "pretend" that it does.

4 Concluding Remarks

We started with two apparently rival positions. One of them (internal realism) claims that there literally are numbers, and that mathematics is literally true. The other (fictionalism) denies both of these claims. Both were suggested as potential responses to the model-theoretic arguments, but on reflection, we found that we could not really tell any difference between the two positions. This leads to some rather striking conclusions in metaontology.

Having rejected metaphysical realism and accepted both Morals (in a particular arena, such as mathematics), many debates in contemporary philosophy (of mathematics) become totally uninteresting. Fictionalists' concerns about "ontological parsimony" (in philosophy of mathematics) are utterly spurious: it is doubtful that internal realists and fictionalists differ in their ontological commitments. Questions

[14]Putnam (1987, p. 17). See also Putnam (1981, p. 55; 1983a, p. xvii; 1983b, p. 84). The extent of Putnam's pragmatism is, though, not altogether clear. For example, immediately after he asserts that "truth is an *idealization* of rational acceptability", Putnam goes on to point out that there is no such thing as ideal rational acceptability. It is simply a useful heuristic, like "the frictionless plane" (Putnam 1981, p. 55).

about whether (mathematical) practice is currently "in error", and whether (mathematical) practitioners are or ought to be realists or fictionalists, turn out to be scarcely well-formed: they simply turn on an inconsequential decision about how to use the word "true" (and, accordingly, "pretence"). The really interesting questions arise from considering the appropriate *theory of truth* (for mathematics), and seeing what impact that theory would have on practice (both mathematical and non-mathematical).[15]

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[15]These conclusions are obviously Dummettian. There are too many relevant Dummett papers to cite; but I take particular inspiration from Dummett (1956; 1959).